

# NUMERICAL SIMULATION OF BUBBLE MOTION IN BOILING NANOFLUIDS BASED ON LATTICE BOLTZMANN METHOD

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### ABSTRACT

A lattice Boltzmann model was established in this paper to analyze bubble flow in boiling nanofluids. Then the bubble growth and departure processes under different superheats and nanopartical concentrations were simulated through the model. The result was compared with that in pure water under the same conditions. The simulation results show that compared with pure water, bubble grew faster with greater frequency of bubble departure, and the bubble departure diameter was slightly larger, then the new fluid was supplemented timely, thus in a time unit, more heat was taken away from the wall, which can explain the heat transfer enhancement by nanofluids. In addition, within a certain range, as the superheat increases, the bubble departure diameter becomes larger.

Keywords: Nanofluids, Lattice Boltzmann Method, departure diameter, heat transfer.

## **1. INSTRUCTIONS**

Nanofluid is a new heat transfer medium characterized by being stable and uniform for solid nanometer-sized particles are added to traditional heat transfer medium such as water, alcohol or oil. It was first proposed by Chois [1] in 1995. As a micro-scale heat transfer, nanofluids boiling heat transfer has been extensively researched at home and abroad. But so far no unified conclusions have been reached as to whether the nanofluids can enhance heat transfer or not [2].

Research that suggests the nanofluids would deteriorate heat transfer includes the following: Cheol et al [3]'s experiment studied the water-based Al<sub>2</sub>O<sub>3</sub> nanofluids with different volume fractions. The conclusion was: whether natural convection or boiling heat transfer, heat transfer ability of nanofluids was always weaker than that of water. Meanwhile Liu et al [4] respectively did similar tests on other types of nanofluids, and a similar conclusion was also reached. Lee et al [5] showed that the nanofluids could strengthen the single-phase heat transfer, but would deteriorate the boiling heat exchanger, probably because of the particle agglomeration resulting in the micro-channel blockage.

Descriptions show that the nanofluids can enhance heat transfer: Hegde et al [6] found that when the volume concentrations were at the range of 0.01% to 0.1%, the alumina nanofluids boiling heat transfer coefficient increased by nearly 48%. Soltani et al [7] studied the influence of  $Al_2O_3$  nanoparticles on the boiling heat transfer characteristics. The results show that: adding  $Al_2O_3$  nanoparticles to the base fluid can improve boiling heat transfer coefficient, and heat capacity will be improved as the concentration increases. The studies of Hyungdae Kim [8] show that the heat transfer performance can be improved by using nanofluids.

Some literatures show that within a certain range, the nanofluids can strengthen heat transfer: Kwark et al [9] studied the boiling heat transfer characteristics with low concentrations of nanoparticles. Results indicate: compared with pure water, when the particle concentration was low, the boiling heat transfer coefficient of nanofluids was basically unchanged. But when the concentration reached a certain value, the boiling heat transfer coefficient decreased significantly. Adirek [10] also conducted nanofluids boiling experiments, and drew the conclusion that in the high heat flux, when the concentrations were below 0.01%, nanofluids can enhance heat transfer, otherwise it would weaken the boiling heat transfer.

Wang et al [11]'s experiments studied the  $Al_2O_3$  nanofluids boiling heat transfer on the smooth surface at different pressures and particle concentrations respectively. Results show that: the concentration of nanoparticals had a significant impact on boiling heat transfer coefficient and the critical heat flux (CHF), besides there was an optimum concentration of particles, which was about 1%.

Through the analysis of the core idea of inhibitory boiling instability, it can be known that the volume expansion caused by bubble growth is the direct reason for flow instability, and that irregular bubble growth would cause irregular boiling heat transfer [12]. So research on the rules and conditions of the bubble growth and departure is of great significance to clarifying the nucleate boiling heat transfer. Existing nucleate boiling heat transfer mechanism models mainly studied a single bubble formation, growth, departure and the accompanying heat transfer phenomenon[13], and all of these mechanistic models contain the bubble parameters to characterize the dynamic process: the departure diameter and departure frequency. In this paper, based on the lattice Boltzmann model presented in literature [14], the bubble growth and departure in the nanofluids are numerically simulated to preliminarily explain the boiling heat transfer of nanofluids.

## 2. LATTICE BOLTZMANN MODEL

In this paper, two macroscopic parameters are introduced: the average number density n and the order parameter  $\phi$ 

$$n = \frac{\rho_A + \rho_B}{2} \quad , \quad \phi = \frac{\rho_A - \rho_B}{2} \tag{1}$$

Where  $\rho_A$  and  $\rho_B$  are the density of fluid A and fluid B respectively. They may be  $\rho_L$  or  $\rho_H$ , which depends on the initial conditions.

When  $\phi > 0$ , the phase-change source term should not be added, the corresponding lattice Boltzmann equation is as follows:

$$g_i(\mathbf{r} + \mathbf{e}_i \delta_i, t + \delta_i) = g_i(\mathbf{r}, t) + (1 - q) \Big[ g_i(\mathbf{r} + \mathbf{e}_i \delta_i, t) - g_i(\mathbf{r}, t) \Big] + \frac{g_i^{eq}(\mathbf{r}, t) - g_i(\mathbf{r}, t)}{\tau_{\phi}}$$
(2)

The lattice Boltzmann implementation of continuity and momentum equations can be described as

$$f_i \left( \mathbf{r} + \mathbf{e}_i \delta_i, t + \delta_i \right) = f_i \left( \mathbf{r}, t \right) + \Omega_i$$
(3)

and

$$\Omega_{i} = -\frac{1}{\tau_{f}} \Big[ f_{\alpha}(\mathbf{r}, t) - f_{\alpha}^{\text{eq}}(\mathbf{r}, t) \Big] + \left( 1 - \frac{1}{2\tau_{f}} \right) \frac{W_{i}}{c_{s}^{2}} \Big[ (\mathbf{e}_{i} - \mathbf{u}) + \frac{(\mathbf{e}_{i} \mathbf{u})}{c_{s}^{2}} \mathbf{e}_{i} \Big] \Big( \mu_{\phi} \vec{\nabla} \phi + \vec{F}_{b} \Big) dt$$
(4)

The simplest distribution function  $h_i$  is as follows

$$\mathbf{h}_{i}\left(\mathbf{r}+\mathbf{e}_{i}\delta_{t},t+\delta_{t}\right)=h_{i}\left(\mathbf{r},t\right)+\frac{h_{i}^{\mathrm{eq}}\left(\mathbf{r},t\right)-h_{i}\left(\mathbf{r},t\right)}{\tau_{T}}$$
(5)

When  $\phi \le 0$ , the phase-change source term should be added into the evolution equations(2)and(5):

$$g_{i}(\mathbf{r}+\mathbf{e}_{i}\delta_{i},t+\delta_{i}) = g_{i}(\mathbf{r},t) + (1-q)\left[g_{i}(\mathbf{r}+\mathbf{e}_{i}\delta_{i},t) - g_{i}(\mathbf{r},t)\right] + \frac{g_{i}^{eq}(\mathbf{r},t) - g_{i}(\mathbf{r},t)}{\tau_{\phi}} + w_{i}\dot{\phi}$$
(6)

$$\mathbf{h}_{i}\left(\mathbf{r}+\mathbf{e}_{i}\delta_{t},t+\delta_{t}\right)=h_{i}\left(\mathbf{r},t\right)+\frac{h_{i}^{\mathrm{eq}}\left(\mathbf{r},t\right)-h_{i}\left(\mathbf{r},t\right)}{\tau_{T}}+w_{i}\frac{\rho_{G}}{\rho_{L}\left(\rho_{L}-\rho_{G}\right)}\frac{\phi}{Ja}$$
(7)

Where

$$\phi = \frac{\rho_L(\rho_L - \rho_G)}{\rho_G} \times \frac{Ja}{Pe} \left(\frac{\partial^2 T}{\partial x^2}\right), Pe = \frac{3}{(\tau_T - 0.5)\delta}, Ja = \frac{1}{h_{fg}} C_{pl} \left(T_b - T_{\infty}\right)$$
(8)

and  $h_{f_8}$  is the latent heat of vaporization.

The equilibrium distribution functions can be constructed as

$$f_{\alpha}^{\text{eq}}(\mathbf{r},t) = w_i A_i + w_i n + \left(3\mathbf{e}_{\alpha} \cdot \mathbf{u} - \frac{3}{2}u^2 + \frac{9}{2}(\mathbf{e}_{\alpha} \cdot \mathbf{u})^2\right)$$
(9)

(Based on D2Q9)

Where the coefficients are

$$A_{1} = \frac{9}{4}n - \frac{15\left(\phi\mu_{\phi} + \frac{1}{3}n\right)}{4}, \quad A_{i}|_{i=2,..,9} = 3\left(\phi\mu_{\phi} + \frac{1}{3}n\right)$$

$$w_{1} = \frac{4}{9}, \quad w_{i}|_{i=2,..,5} = \frac{1}{9}, \quad w_{i}|_{i=6,..,9} = \frac{1}{36}$$
(10)

Where the chemical potential is

$$\mu_{\phi} = A \left( 4\phi^3 - 4\phi^{*2}\phi \right) - k\nabla^2 \phi \tag{11}$$

In which 
$$A = \frac{3\sigma}{4W\phi^{*4}}$$
,  $k = \frac{1}{2}A(W\phi^{*})^{2}$  (12)

Where  $\sigma$  and W are the surface tension and the thickness of the interface layer respectively.  $\phi^*$  is the expected order parameters,  $\phi^* = (\rho_{\mu} - \rho_l)/2$ .

$$g_{\alpha}^{eq}(\mathbf{r},t) = A_i + B_i \phi + C_i \phi \mathbf{e}_i \cdot \mathbf{u}$$
(13)

(Based on D2Q5)

The coefficients can be chosen as

$$A_{i} = -2\Gamma\mu_{\phi}, \quad A_{i} = \frac{1}{2}\Gamma\mu_{\phi} (i \neq 1)$$
  

$$B_{i} = 1, \quad B_{i} = 0 \ (i \neq 1), C_{i} = \frac{1}{2q}$$
(14)

Where  $\Gamma$  is used to control the mobility.

Т

The equilibrium distribution functions of temperature:

$$a = \sum f_a^{eq}(\mathbf{r}, t) \tag{15}$$

$$=\sum h_{\alpha}^{\rm eq}(\mathbf{r},t) \tag{16}$$

$$\mathbf{u} = \left(\sum_{i} f_{a}^{eq} \left(\mathbf{r}, t\right) \mathbf{e}_{i} + \frac{1}{2} \left(\mu_{\phi} \vec{\nabla} \phi + \vec{F}_{b}\right)\right) / n$$
(17)

$$\sum_{i} f_{\alpha}^{\text{eq}}(\mathbf{r}, t) e_{i\alpha} e_{i\beta} = \left( \phi \mu_{\phi} + c_{s}^{2} n \right) \delta_{\alpha\beta} + n u_{\alpha} u_{\beta}$$
<sup>(18)</sup>

$$\phi = \sum_{i} g_{i} = \sum_{i} g_{\alpha}^{eq}(\mathbf{r}, t)$$
(19)

$$\frac{\phi}{q}u_a = \sum_i g_a^{\rm eq}(\mathbf{r}, t) \mathbf{e}_{ia}$$
<sup>(20)</sup>

$$\sum_{i} g_{\alpha}^{eq}(\mathbf{r}, t) e_{i\alpha} e_{i\beta} = \Xi_{\alpha\beta}$$
(21)

Where 
$$\Xi_{\alpha\beta} = \Gamma \mu_{\phi} \delta_{\alpha\beta}, \ q = 1/(\tau_{\phi} + 0.5)$$
 (22)

The bubble motion is described by above evolution equations and equilibrium distribution functions.

#### **3. PHYSICAL PARAMETERS OF NANOFLUIDS**

In this paper, the subscripts of f, nf, p mean basefluid, nanofluid and particle, respectively.

Table1. Physical parameters of water and Al<sub>2</sub>O<sub>3</sub>

parameters	Water	Al <sub>2</sub> O <sub>3</sub>	
ρ (kg/m3)	997.1	3970	
cp (J/kg k)	4174	765	
$v_{(m2/s)}$	0.001004	/	
k(Wm-1 K-1)	0.618	25	
$\sigma_{ m (10-3N/m)}$	69		

The physical parameters of nanofluids can be calculated as follows:

The density of nanofluids is calculated as [16]

$$\rho_{nf} = (1 - \theta)\rho_f + \theta\rho_p \tag{23}$$

The specific heat is obtained as [16]

$$\rho_{nf}C_{pnf} = (1-\theta)\rho_f C_{pf} + \theta\rho_p C_{pp}$$
(24)

The viscosity of nanofluids [17] is

$$\mu_{\rm ef} / \mu_{\rm f} = 0.904 e^{14.8\theta} \tag{25}$$

The heat transfer coefficient of nanofluids can be calculated as [19]

$$\frac{k_{nf}}{k_{f}} = \frac{k_{p} + 2k_{f} - 2\theta \left(k_{f} - k_{p}\right)}{k_{p} + 2k_{f} + \theta \left(k_{f} - k_{p}\right)}$$
(26)

The thermal diffusion coefficient of nanofluids[18] is

$$a_{nf} = k_{nf} / \left(\rho c_p\right)_{nf} \tag{27}$$

The surface tension of nanofluids [19] is

$$\frac{\sigma_f - \sigma_{nf}}{\sigma_f} = b \ln(\frac{\theta}{a} + 1)$$
(28)

For Al2O3-water nanofluids,  $a = 7.673 \times 10^{-7}$ ,  $b = -7.773 \times 10^{-3}$ Besides, the latent heat of Al<sub>2</sub>O<sub>3</sub>-water nanofluids with the particle size of 40nm are shown as below.

 Table 2. Latent heat of Al2O3-water nanofluids with different concentrations [20]

Concentratio $_{\rm n} \theta$	0vol.	0.1vol.	0.2vol.	0.4vol.	0.5
	%	%	%	%	vol.%
h <sub>fg</sub> (J/g)	1991	1947	1929	1882	1850

In this paper, unit lattice length  $L=1\times 10^4 m$ , and a unit of evolution time  $t=6\times 10^4 s$ , the dimensionless temperature  $\Theta = (T-T_{in})/(T_w-T_{in})$ , and the departure diameter is calculated by:  $D = \sqrt{4A/\pi}$ , where A is the area of bubbles.

## 4. RESULTS AND ANALYSIS

The nucleate boiling simulations are carried out in a rectangular domain whose mesh is generated as 100\*60. The superheat is set as  $\Delta T = 1.5$  K, and the center of a spherical

bubble with the radius of two in lattice units is located at (50, 0). The physical model adopted in this article is shown in Figure 1, the green particles represent pure water or nanofluids, red for the gas-liquid interface, and the bottom is the heating surface.

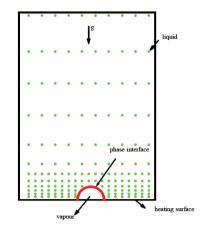


Figure 1. The physical model at initial time

From Figure 2, it can be seen that the results calculated by LBM is well consistent with the experimental results of the Kim [21], which indicate that it is reliable to simulate the behavior of bubbles in boiling fluid by LBM.

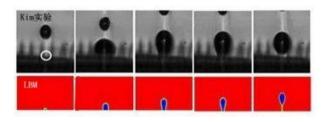
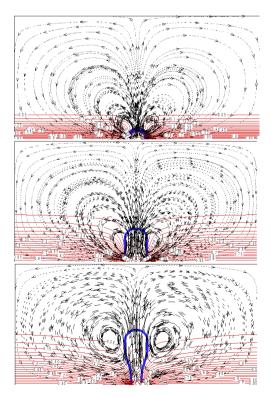
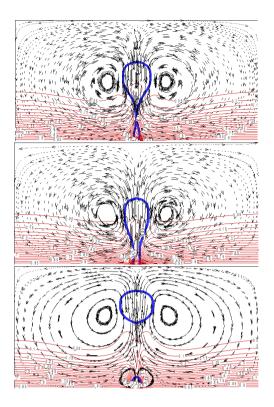


Figure 2. The Comparison of LBM with Kim's experiment





**Figure 3.** The topological structure,temperature field and flow field of a single bubble

Figure 3 shows a typical cycle of the single bubble topological structure, temperature field and flow field, in which the black line is the flow field line, and the red line is the contour temperature field. As can be seen from the figure, in the initial stages of the bubble growth, due to the high heat flux continuously introduced by the wall, as time goes on, the bubble continues to grow. When the bubble diameter grows to a certain extent, a neck is formed in the distance of the wall. This neck is very important, since the bubble is separated from the wall here after fracture. While the formed small bubble in the low part still stays in the wall, the up bubble would shrink rapidly and move upward, due to the effect of surface tension and gravity. In addition, it can also be seen from the figure that there are two symmetrical swirls around the bubble, and the swirls play an important role in transporting the cold fluid to the bubble neck. It is similar with the transient convection model proposed by Haider et al [22]: before the bubble departures from the wall, the bubble becomes larger and larger due to the heat flux continuously introduced from the wall. In addition, the gravity and the force between wall and fluid also act on the bubble. As a result, along with the upward movement, the bubble becomes larger, and the hot fluid near the gas-liquid interface will also move upward with the bubble, so the cold fluid will be driven to the bubble neck, resulting in two symmetrical on both sides of the vortex bubble. It can also be seen from the figure that with the rising of the bubble, the temperature of the computational domain will also be significantly increased.

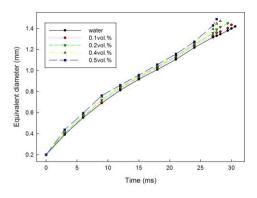


Figure 4. The bubble diameter in the boiling process

Fig. 4 shows the change of the bubble diameter over time in the fluid boiling process under the condition that  $\Delta T=1.5K$ . It can be seen from the figure that under the same superheat, the bubble departure diameter in nanofluid is larger than that in pure water, and the bubble departure time in nanofluid is slightly shorter than that in pure water. Besides, with the increase of the concentration of nanoparticles, the departure diameter becomes larger, and the departure time will be shorter. It indicates that in a time unit, more heat is taken away by the bubble, and the boiling process is stronger. This is mainly because in a relatively small range of nanoparticles concentration, increasing the concentration of nanoparticles can effectively improve the thermal conductivity of nanofluids and the ability of heat exchange, rendering the vaporization of overheating layer close to the heated wall faster. As a result, in the same equivalent bubble diameter, it makes the contact surface between the wall and the bubble smaller, the buoyancy larger, so that the bubble grows faster and earlier away from the heating surface. In addition, the fact that bubble grows faster will make the inertial force bigger, so the departure equivalent diameter is larger.

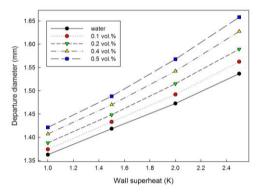


Figure 5. The departure diameter under different superheats

Figure.5 shows the change of bubble departure diameter with different superheats under different concentrations. It can be seen from the figure that under the same superheat, the higher the concentrations, the larger the departure diameter.

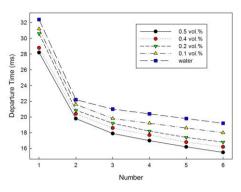


Figure 6. The bubble departure time under the condition  $\triangle T = 1.5 K$ 

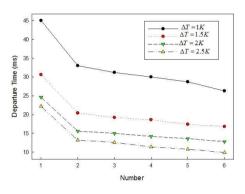


Figure 7. The departure time under different superheats with the concentration of 0.4 vol. %

It can be seen from Figure.6 that under the same wall superheat, compared with the pure water, the bubble departure frequency in nanofluids is greater, and the boiling process is fiercer. Another conclusion can also be reached from Figure.7 that under the same concentration, as the superheat increases, the bubble departure period will be reduced. This is because with the first bubble getting away from the wall, the second small bubble below the bubble neck will be formed at the wall. After the bubble departure, due to gravity and surface tension, the bubble will move upward, which will cause the thermal fluid near the gas-liquid interface to flow upward with the bubble, and the cold fluid to move downward. Thus the heat transfer and gas around the nucleus is strengthened, so that the second bubble departure time is much shorter than the first. When the second bubble detaches from the wall, the third small bubble will be formed below the neck. Over time, the bubble will continue to grow and detach and the cycle continues. The thermal diffusivity of nanofluids is higher than that of water, and it will increase with the increase of the concentration, so that more heat will be taken away by bubbles.

## **5. CONCLUSION**

In this paper, a lattice Boltzmann model was established to simulate the process of bubble growth and departure with different concentrations and superheats. Conclusions can be obtained as follows:

(1) Under the same superheat, the bubble departure time, frequency and the equivalent diameter will be shorter, larger and bigger respectively, along with the increase of concentrations.

(2) At the same concentration, when the superheat increases, the bubble growth process accelerates and the frequency increases.

To sum up, under the same condition, compared with pure water, the bubble departure time in nanofluids is shorter, the frequency is higher, and the departure diameter is slightly larger. In addition, with the increase of concentration of the nanoparticles, in a time unit, more heat will be taken away by the bubble, which indicates that in a certain range, boiling heat transfer can be enhanced by nanofluids. This article can preliminarily explain the reason why the nanofluids could enhance the heat transfer from the perspective of bubble dynamics.

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