it can be seen that an increment in *Le* enlarges the temperature of nanofluid close to the plate so little and reduces the temperature in the farther distance from the surface. As indicated in Figure 7, thermal boundary layer thickness and temperature, both enhance with the increment of *Nb* and *Nt*. The extra heating generated by interaction between the nanoparticles and host fluid as a result of the Brownian movements and thermophoresis influences augments the temperature and the thermal boundary layer thickness.

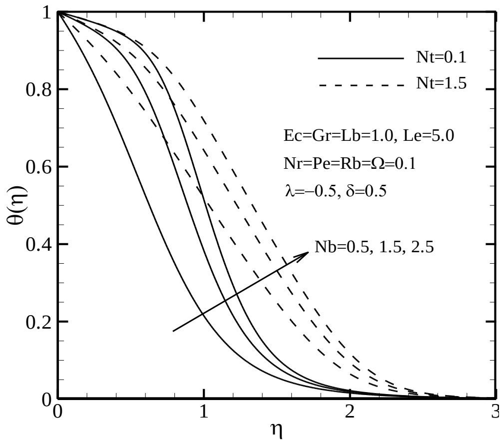

Figure 7. The effects of *Nt* and *Nb* on temperature

As clearly shown in Figure 8, an increment in *Ec* boosts boundary layer thickness of the nanoparticles concentration as the nanoparticles concentration reduces. This figure also illustrates that λ<0 and λ>0 make the decrease and increase of nanoparticles concentration near the sheet, respectively. As pointed out earlier, the velocity augments with *Ec* and λ>0 which causes the decrement of nanoparticles concentration.

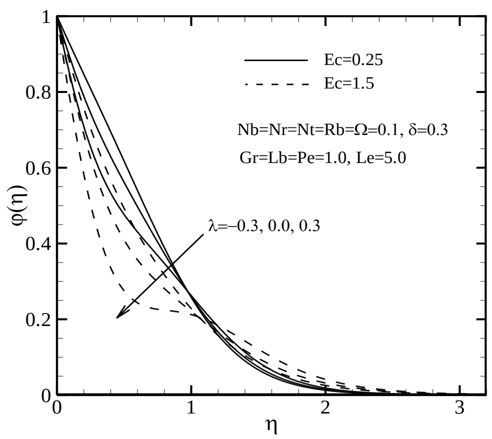

Figure 8. The effects of *Ec* and λ on nanoparticles concentration

Figure 9 illustrates the effects of *δ* and *Le* on nanoparticles distribution. As *Le* enhances, the concentration and the thickness of boundary layer extremely decrease. To justify this observation, it should be pointed out that as *Le* increases, the convection of nanoparticles increases against the mass diffusion and Brownian motion, hence; both of the concentration and the nanoparticles boundary layer thickness decreases. The nanofluid forced convection decreases as *δ* increases, therefore it is perspicuous the nanoparticles concentration is increased by increasing *δ*. Increasing *Nt*, increases both of the concentration and nanoparticle concentration layer thickness, whereas increment of *Nb* has the reverse effect of declining on both of them as is shown in Figure 10.

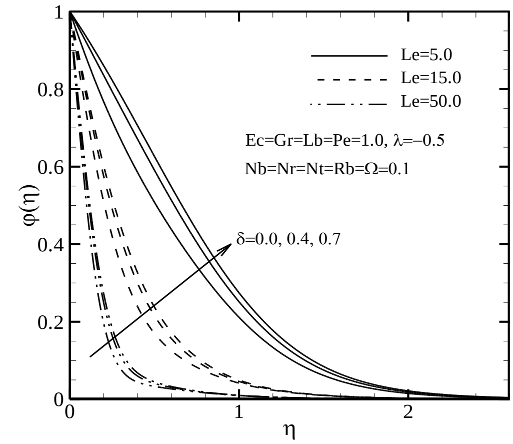

Figure 9. The effects of *Le* and *δ* on nanoparticles concentration

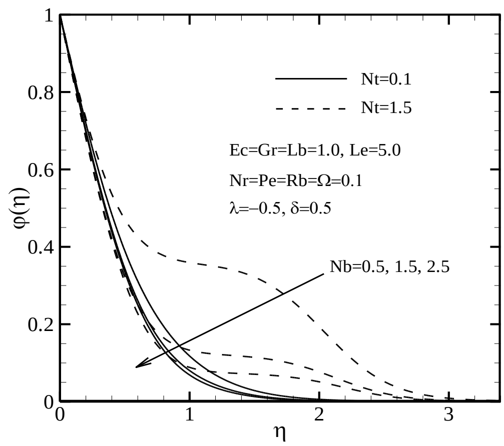

Figure 10. The effects of *Nb* and *Nt* on nanoparticles concentration

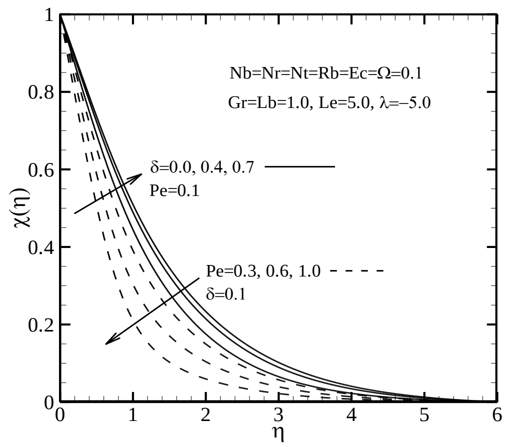

Figure 11. The effects of *δ* and *Pe* on density of microorganisms

The effects of different pertinent variables on the non-dimensional density of microorganisms are presented in Figures 11-13. It can be observed increasing *Pe*, *Lb*, *Gr* and *Ω* decreases both the dimensionless density of motile microorganisms and its thickness, whereas *δ* makes the increase of the dimensionless density of motile microorganisms within boundary layer.

Figure 12 also indicated that λ<0 and λ>0 decreases and increases the density of microorganisms, respectively. It is noteworthy that λ has not a significant influence on the thickness of boundary layer for motile microorganisms.

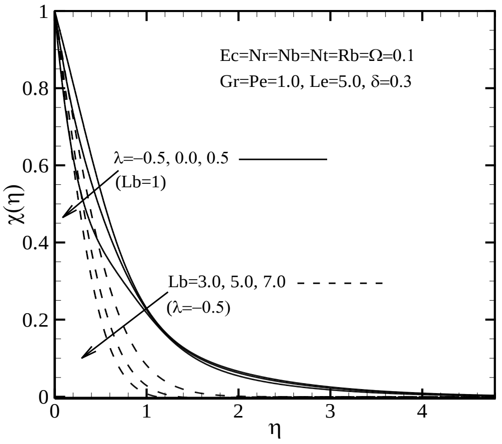

Figure 12. The effects of λ and *Lb* on density of microorganism

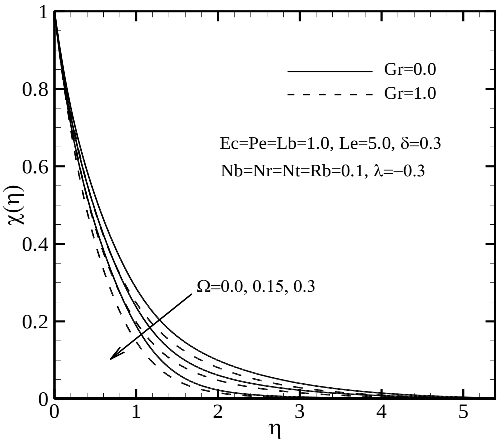

Figure 13. The effects of *Ω* and *Gr* on density of microorganisms

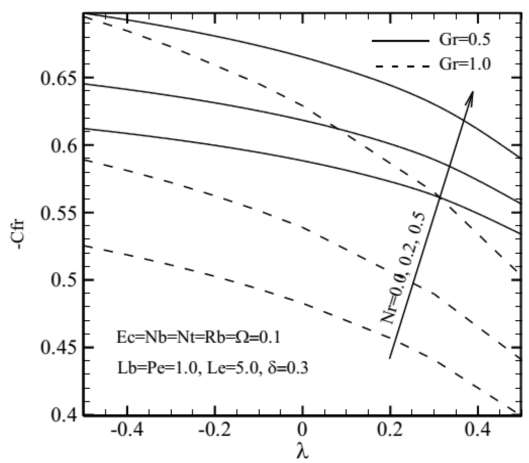

Figure 14. The effects of λ, *Gr* and *Nr* on the reduced skin friction coefficient

The diffusion of microorganisms decrease with enhancing *Pe* and *Lb*, hence, it is envisaged that both of density and thickness of boundary layer for microorganisms decreases with an increase in *Pe* and *Lb*. Also, it can be stated that decreasing the fluid velocity due to the presence of *δ* and λ>0 enhances the dimensionless density of microorganisms. In addition, there is a reverse observation when λ<0. The convection caused by the buoyancy force rises with increasing *Gr* and as a result the density number of the microorganisms declines with an increment in *Gr*.

An increasing in *Gr* and *Nr* decreases and increases *Cfr*, respectively. Also, it can be observed that the influence of *λ* on the reduction or enhancement of *Cfr* increases as *Gr* amplifies. The variations of *Cfr* with δ, *Ec* and *Le* are pictured in Figure 15. As observed, an increasing in *Le*, *Ec* and *δ* causes the declines of *Cfr*.

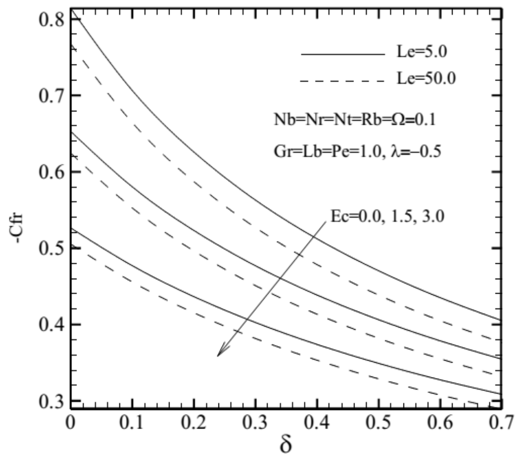

Figure 15. The effect of *δ*, *Le* and *Ec* on the reduced skin friction coefficient

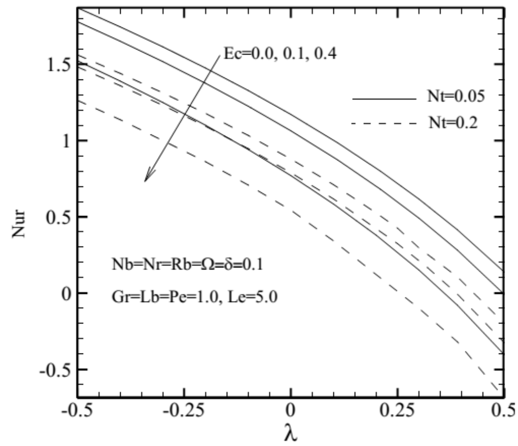

Figure 16. The effect of λ, *Ec* and *Nt* on the reduced Nusselt number

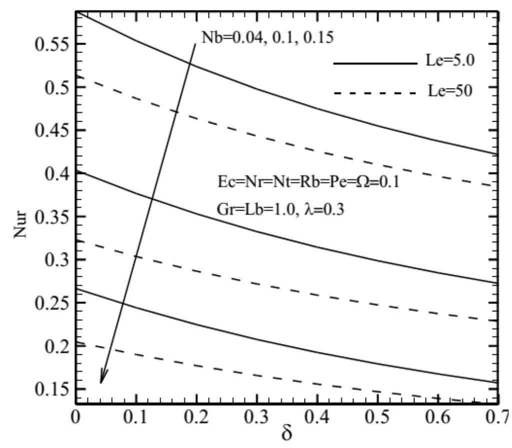

Figure 17. The effect of δ, *Le* and *Nb* on the reduced Nusselt number

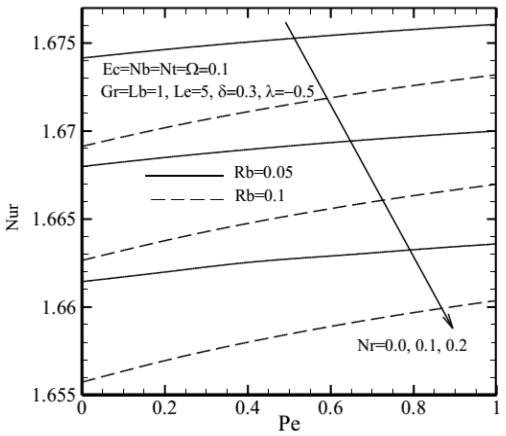

Figure 18. The effect of *Pe, Nr* and *Rb* on the reduced Nusselt number

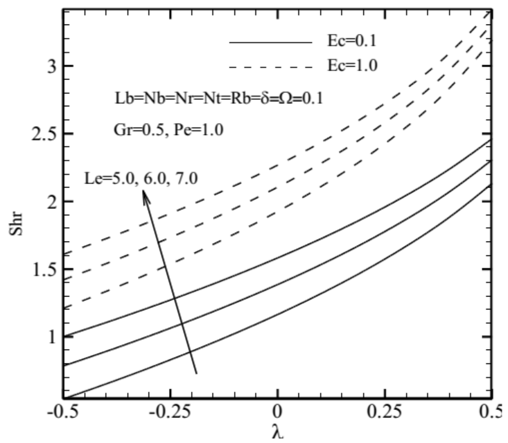

Figure 19. The effect of λ, *Ec* and *Le* on the reduced Sherwood number

As depicted in Figures 16 and 17, *Nur* the representative of heat transfer rate, diminishes with increasing *Ec*, *Nt*, *Nb*, *δ*, *Le* and λ>0, while increases with λ<0.

As noted in Figures 4-7, the fluid temperature in the boundary layer increases with increment of *Ec*, *Nt*, *Nb*, δ, *Le* and λ>0 and decreases with λ<0 and as a result *Nur* decreases with *Ec*, *Nt*, *Nb*, δ, *Le* and λ>0 and increases with λ<0. Fig. 18 illustrates that the increase of *Pe* rises the heat transfer rate at the sheet, whereas an increment in *Nr* and *Rb* decline reduced Nusselt number *Nur*. Indeed, this is for the dimensionless temperature enhances because of the decrease of velocity with *Rb* and *Nr*.

The variations of reduced Sherwood number *Shr* with various parameters are illustrated in Figures 19 and 20. It is obvious that *Shr* increases with an increasing in *Ec*, *Le*, *Nb* and λ>0 and decreases with *Nt*, *δ* and λ<0. The influences of the different variables on the reduced density number of the motile microorganisms *Nnr* are displayed in Figures 21 and 22. As it is shown, *Nnr* increases with an increasing in *Gr*, *Lb*, *Pe*, Ω and λ>0 while an increasing in δ and λ<0 causes the decrease of *Cfr*. As mentioned before, the augmentation of *Lb* and *Pe* amplifies the convection of motile microorganisms against their diffusion, accordingly, the microorganisms concentration reduces and the concentration gradient rises near the flat plate with increasing *Lb* and *Pe*. Also was said that *δ* and λ<0 reduce the dimensionless velocity and increase the density of motile microorganisms in boundary layer. Consequently, *Cfr* decreases with an increase in *δ* and λ<0.

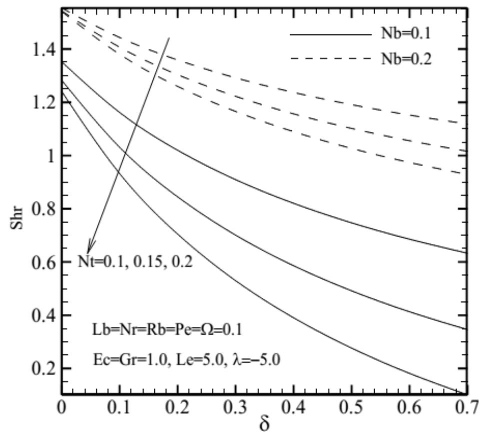

Figure 20. The effects of *δ*, *Nb* and *Nt* on the reduced Sherwood number

Increasing the dimensionless velocity in boundary layer due to enhancement *Gr* diminishes the density of motile microorganisms and as a result increases the motile microorganisms transfer rate from the stretching plate.

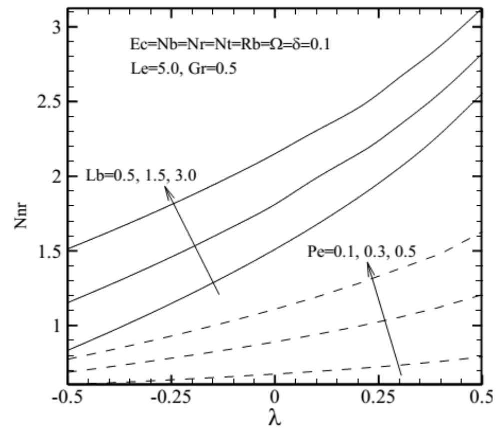

Figure 21. The effects of λ, *Lb* and *Pe* on the reduced density number of the motile microorganism

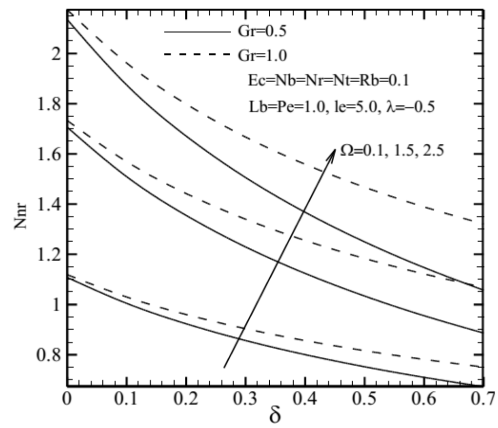

Figure 22. The effects of δ, *Ω* and *Gr* on the reduced densitynumber of the motile microorganisms

5. CONCLUSION

Behavior of a nanofluid consisting of gyrotactic microorganisms passing a stretching sheet with considering the impacts of heat absorption/generation and partial slip at the surface is numerically conducted in current investigation. The governing partial differential equations are shifted to a system of the non-linear, coupled and ordinary differential equations using transformations a system of similarity and are solved by the finite difference method with linearization technique of Newton. Numerical results of velocity, temperature, concentration of nanoparticles and density of microorganisms as well as reduced skin friction coefficient Cfr, reduced Nusselt number Nur, reduced Sherwood number Shr and reduced density number of the microorganisms Nnr are represented and discussed in detail. The major findings of this study are outlined below:

I. The dimensionless velocity of nanofluid increases with the increment in Ec, Gr and $\lambda>0$ while reduces with δ, Rb, Nr and $\lambda<0$. Also, it is found that these parameters have no impact on the thickness of momentum boundary layer.

II. The dimensionless temperature of nanofluid increases as Ec, Nb, Nt, δ and $\lambda>0$ increases and decreases in the presence of $\lambda<0$. Also, it is seen that as Le increases the temperature increases near the stretching plate so little and reduces in the farther distance from the surface.

III. The nanoparticles concentration increases as δ, Nt and $\lambda<0$ increase while decreases with the increase of Nb, Le, Ec and $\lambda>0$. An increase in Le and Nt extremely enhance the concentration boundary layer thickness. Additionally, it is observed that the nanoparticles concentration boundary layer thickness slightly boosts with an enhancing in Ec.

IV. The dimensionless density of motile microorganisms increases as δ and $\lambda>0$ increase while decreases as Pe, Lb, Gr, Ω and $\lambda<0$ increase. It is also observed that thickness of boundary layer for motile microorganisms declines with Lb and Ω.

V. As Gr, Le, Ec, δ and $\lambda>0$ increase, the reduced skin friction coefficient decreases while augments with the increment of Nr and $\lambda<0$.

VI. The reduced Nusselt Nur decreases with increasing Ec, Nt, Nb, δ, Le, Rb, Nr and $\lambda>0$, while increases with Pe and $\lambda<0$.

VII. An increasing in Le, Ec, Nb and $\lambda>0$ cause the enhancement of the reduced Sherwood Shr while increase in δ, Nt and $\lambda<0$ cause the reduction of Shr.

The reduced density of the motile microorganisms Nnr increases as Gr, Ω, Lb, Pe and $\lambda>0$ increase and decreases as δ and $\lambda<0$ increase.

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