On Development of Analytical Approach for Analysis of Energy Transfer of Traveling Wave Tube

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ABSTRACT

In this paper we introduce an analytical approach for analysis of operating regimes of traveling wave type tube O. Based on this approach we analyzed the energy transfer of the concerned tube. Based on this analysis we formulate conditions to increase efficiency of this traveling wave tube.

1. INTRODUCTION

At the present time, intensive development of sources of electromagnetic radiation (gyrotrons, ortrons, traveling wave tube, power amplifiers also develop) could be found [1-5]. It is attracted an interests increasing power of emitting of electromagnetic radiation, increasing of electronic efficiency, decreasing of dimensions of devices, development of mathematical approaches to improve prognosis of operating of the considered devices [6-15]. In this paper we introduce an analytical approach for analysis of processes in traveling wave tube type O and determination of operation conditions of the tube (see Figure 1). Based on this approach, the energy transfer of the considered traveling wave tube and the possibility of increasing its efficiency were analyzed.

2. METHOD OF SOLUTION

For analysis of processes taking place in traveling wave tube, the greatest interest is in the study of the motion of ultrarelativistic electrons in a traveling wave field with a constant amplitude, which we shall consider equal to

\[ E_z = E_0 \sin(k_z z - \omega t), \]  

where \( z \) is the coordinate, \( k_z \) is the projection of the wave number on the Oz axis, \( t \) is the time, \( \omega \) is the radiation frequency, and \( E_0 \) is the field amplitude. At relativistic electron velocities, it is more convenient to use the relative electron energy \( \gamma \), equal to the ratio of the electron energy \( \varphi = mc^2 \) to the rest energy \( \varphi_0 = m_0 c^2 \), \( m_0 \) is the rest mass. From the law of conservation of energy

\[ \gamma = 1 + \frac{eU}{m_0 c^2}, \]

where \( e \) is the electron charge, and \( U \) is the accelerating potential difference. The change in the electron energy under the action of the field in this case is described by the following equation

\[ \frac{d \gamma}{d z} = \frac{e}{m_0 c^2} \frac{d U}{d z} = -\frac{e E_z}{m_0 c^2} \sin(k_z z - \omega t). \]

We introduce a new variable: the phase of the electrons with respect to the radio-frequency field \( \vartheta = k_z z - \omega t \). The equation for describing the phase is represented in the following form

\[ \frac{d \vartheta}{d z} = k_z - \omega \frac{d t}{d z} = k_z - \frac{\omega}{v_\parallel}. \]
As a result, to describe the transfer of energy to the traveling wave tube, we obtain two equations (3) and (4). We assume that the electron energy is \( \gamma = \gamma_0(1+u) \), where \( \gamma_0 \) is the energy of the particle at the entrance to the system, \( u \) is the relative change in energy under the action of the field. We express \( v_\parallel \) through \( \gamma_0 \) and \( u \)

\[
v_\parallel = c \sqrt{1 - \gamma^{-2}} = c \sqrt{1 - \gamma_0^{-2}(1+u)^{-2}}.
\]

The relative change in energy under the action of the field is often small. In this case we can assume that

\[
v_\parallel \approx c \sqrt{1 - \frac{2u}{\gamma_0^2}} = c \sqrt{k_z^2 + \frac{2u}{\gamma_0^2}} \approx c k_z \left(1 + \frac{u}{\gamma_0^2 k_z^2}\right).
\]

Substitution of this relation into equation (4) leads to the following result

\[
\frac{d \vartheta}{dz} = k_z - \frac{\omega}{c k_z (1 + u \gamma_0^2 k_z^{-2})} \approx k_z - \frac{\omega}{v_\parallel} \left(1 - \frac{u}{\gamma_0^2 k_z^2}\right) = k_z - \frac{\omega}{v_\parallel} + \frac{\omega u}{\gamma_0^2 k_z^2 v_\parallel}.
\]

We transform the last equation so that the coefficient of \( u \) is equal to unity and introduce the dimensionless length \( \xi = \omega k_z \gamma_0^2 c \) and the dimensionless detuning of the synchronism \( \Delta = (k_z - \omega v_\parallel) \gamma_0^2 k_z \omega \). Then the equation for the phase was reduced to the following form

\[
\frac{d \vartheta}{d \xi} = u \Delta. \tag{4a}
\]

Substituting the relation \( \gamma = \gamma_0(1+u) \) into relation (3), we obtain

\[
\frac{d u}{d \xi} = F \sin(\vartheta). \tag{5}
\]

where \( F = E_0 \gamma_0 \omega m c \), \( u \) is the relative change in energy under influence of field. The latter equations are supplemented by boundary conditions. We assume that the electron beam at the input \( (\xi=0) \) is stationary and monoenergetic. Then the boundary conditions are written in the form

\[
u(0) = 0; \ \vartheta(0) = \vartheta_0, \ \vartheta_0 \in [0,2\pi]. \tag{6}
\]

Let us now find the expression for the electronic efficiency \( \eta \). The latter can be defined as the ratio of the energy increment averaged over the initial phases \( \vartheta_0 \) to the original electron energy

\[
\eta(\xi) = \frac{\varepsilon(\xi) - \varepsilon(0)}{\varepsilon(0)}. \tag{7a}
\]

The last relation, taking into account the relation \( \gamma = \gamma_0(1+u) \), takes the following form

\[
\eta(\xi) = \frac{2}{2\pi} \int_{\vartheta_0}^{\vartheta_0 + \pi} u(\xi) d \vartheta_0. \tag{7b}
\]

3. DISCUSSION

In this section we analyze the dependence of the electronic efficiency on several parameters. Figure 2 shows the dependence of the efficiency on the relative change in energy under the action of the field. The figure shows monotonous increasing of this dependence. In Figure 3 shows the dependence of the efficiency on the dimensionless length with explicit maximum.

![Figure 2](image1.png)

**Figure 2.** Dependence of the efficiency on the relative change in energy under the action of the field

![Figure 3](image2.png)

**Figure 3.** Dependence of the efficiency of a dimensionless length

![Figure 4](image3.png)

**Figure 4.** Dependence of relative electron energy \( \gamma \) on accelerating potential difference \( U \)
The Figure 4 and 5 show dependence of relative electron energy on accelerating potential difference $U$ and rest energy $\varepsilon_0$. These figures show linear increasing and hyperbolic decreasing of the considered relative electron energy. Both dependences are enough natural.

**Figure 5.** Dependence of relative electron energy $\gamma$ on rest energy $\varepsilon_0$

Velocity $v_\parallel$ increases with increasing of the above relative electron energy $\gamma$ (see Figure 6). Analogous dependence could be found as a function of the energy on relative changing in energy under the action of field $u$ (see Figure 7).

**Figure 6.** Dependence of velocity $v_\parallel$ on relative electron energy $\gamma$

**Figure 7.** Dependence of velocity $v_\parallel$ on relative changing in energy under the action of field $u$

Dimensionless detuning of the synchronism $\Delta$ decreasing at absolute value with increasing of wave number $k_z$ (see Figure 8), leads to the constant value with increasing of frequency $\omega$ (see Figure 9), decreasing at absolute value with decreasing of velocity $v_\parallel$ (see Figure 10).

**Figure 8.** Dependence of dimensionless detuning of the synchronism $\Delta$ on wave number $k_z$

**Figure 9.** Dependence of dimensionless detuning of the synchronism $\Delta$ on frequency $\omega$

**Figure 10.** Dependence of dimensionless detuning of the synchronism $\Delta$ on velocity $v_\parallel$

Dimensionless length $\xi$ increases with increasing of frequency $\omega$ became also unlimited with linear increasing (see Figure 11) and with cubic nonlinearity as function of wave number $k_z$ (see Figure 12).
Phase of the considered electrons with respect to the radio-frequency field $\vartheta$ has linear increasing with increasing of wave number $k_z$ (see Figure 12) and linear increasing with increasing of frequency $\omega$ (see Figure 13).

4. CONCLUSION

In this paper we introduce an analytical approach for analysis of operating regimes of traveling wave tube type O. Based on this approach we analyzed the energy transfer of the considered tube. Based on this analysis we formulate conditions to increase efficiency of this traveling wave tube. For example, efficiency of traveling wave tube could be increased with increasing of energy of electromagnetic field. Efficiency of this tube also depends on its length. But this dependence is nonmonotonous and has a single maximal value.

REFERENCES


