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Thermal Expansion Behavior of Prefabricated Box Culverts and Its Impact on Structural Stability



Lifang Chen^{1*}, Ruibao Jin²

¹ College of Civil Engineering, Xuchang University, Xuchang 461000, China ² Henan Jiaotou Jiaozheng Expressway Co., Ltd, Zhengzhou 450000, China

Corresponding Author Email: 22016006@xcu.edu.cn

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ABSTRACT

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Prefabricated box culverts, recognized as efficient and high-quality precast concrete structures, play a pivotal role in modern urban infrastructure. However, their practical application is often subject to complex thermal environments, particularly the impact of thermal expansion on structural integrity and stability, which has not been thoroughly investigated. This study delves into the thermal expansion behavior of prefabricated box culverts under three-dimensional stress conditions, examining the accuracy of thermal expansion coefficients. A temperature-stress coupled model has been developed to systematically analyze the variation of thermal stress under different temperature conditions and its effect on structural stability. It has been discovered that the triaxial linear thermal expansion strain, average linear thermal expansion coefficient, and thermal expansion stress of prefabricated box culverts are significantly influenced by the threedimensional stress state, a complexity not effectively reflected in simplified models present in existing literature. Through experimental data and comparative analysis, this research validates the theoretical model of thermal expansion coefficients under three-dimensional stress conditions. Additionally, the temperature-stress coupled analysis reveals the thermal stress distribution characteristics of prefabricated box culverts under various temperature gradients, offering new theoretical foundations and analytical tools for the design and safety assessment of precast box culvert specimens. The findings of this study provide a novel perspective on the behavior of prefabricated box culverts in service environments, playing a significant role in ensuring their safe operational performance. Moreover, the proposed theoretical models and analytical methods can serve as a reference for the application of other types of precast concrete structures in similar environments.

1. INTRODUCTION

With the rapid advancement of infrastructure development, prefabricated box culverts, as a novel type of precast concrete structure, have been widely applied in urban drainage systems, underground passages, and other engineering projects, owing to their high construction efficiency and easily controllable quality [1-5]. However, in practical engineering applications, these culvert structures often face complex thermal environments. The thermal expansion behavior, in particular, plays a decisive role in the integrity and stability of the structures [6-8]. Therefore, an in-depth study of the thermal expansion behavior of prefabricated box culverts is of significant importance to ensure their reliability and safety under various environmental conditions.

Although existing research has explored the thermal expansion properties of precast concrete components, there is a scarcity of literature focused specifically on prefabricated box culverts [9-11]. The interaction between the temperature and stress fields in these culverts is complex, with general temperature changes potentially causing uneven thermal expansion, thereby affecting structural stability [12-15].

Consequently, research on the thermal expansion behavior of prefabricated box culverts and its impact on structural stability is both urgent and crucial.

Existing studies have primarily focused on the thermal expansion effects under single conditions or have used simplified models for analysis [16-18], largely overlooking the complex three-dimensional stress states that prefabricated box culverts experience in actual applications and the influence of such states on thermal expansion behavior [19-22]. Moreover, current research methods often lack precision in predicting and validating thermal expansion coefficients, which, to a certain extent, limits the application of theoretical research findings in engineering practice.

This article systematically tests and validates the thermal expansion behavior of prefabricated box culverts. Initially, the triaxial linear thermal expansion strain, average linear thermal expansion coefficient, and thermal expansion stress of the culverts under three-dimensional stress states are determined experimentally. The experimental results are then compared and analyzed against theoretical predictions to validate the accuracy of the thermal expansion coefficients under threedimensional stress conditions. Subsequently, a temperaturestress coupling model is constructed to analyze in-depth the variation of thermal stress in prefabricated box culverts under different temperature conditions and its impact on structural stability. This research not only extends the design theory of prefabricated box culverts but also provides a reference for the safety analysis of other precast concrete structures in complex environments, possessing significant academic value and practical significance.

2. TESTING AND VALIDATION OF THERMAL EXPANSION BEHAVIOR IN PREFABRICATED BOX CULVERTS

2.1 Testing of thermal expansion behavior

This study conducts rigorous testing and validation of the thermal expansion behavior of prefabricated box culvert specimens in actual engineering applications. By testing thermal expansion strain and measuring thermal expansion stress under three-dimensional stress states, this study reveals the true values of thermal expansion coefficients and further validates the applicability and accuracy of theoretical predictions under complex loading conditions. These findings assist engineers in more accurately predicting the performance of prefabricated box culverts under the influence of temperature changes during design and assessment, thereby optimizing structural design, reducing adverse effects caused by temperature, and enhancing the safety and durability of the structures.



Figure 1. The longitudinal sensor placement in prefabricated box culverts

Prefabricated box culverts in real service environments are typically subjected to complex three-dimensional stress fields. This complex stress state has a decisive impact on the thermal expansion behavior of the culverts, and conventional onedimensional or two-dimensional thermal expansion tests cannot fully reflect the structural response under actual working conditions. Therefore, this study initiates the testing of thermal expansion behavior by determining the triaxial linear thermal expansion strain, average linear thermal expansion coefficient, and thermal expansion stress of prefabricated box culvert specimens under three-dimensional stress states. The testing of triaxial linear thermal expansion strain provides actual performance data for the culverts in different directions, offering data for assessing strain distribution in heterogeneous temperature fields. Determining the average linear thermal expansion coefficient helps understand the overall thermal expansion characteristics of the culverts under combined stress influences. Accurate measurement of thermal expansion stress is crucial for assessing the stability of the structure under temperature

variations. These three aspects of testing mutually validate and complement each other, providing a comprehensive description of thermal expansion behavior. Figure 1 presents a schematic diagram of the longitudinal sensor placement in prefabricated box culverts.



Figure 2. Side cross-sectional load condition of prefabricated box culvert specimens

In this study, the thermal expansion behavior of prefabricated box culvert specimens under actual engineering conditions is tested and validated. Initially, the specimens are placed in a test environment that simulates real service conditions and subjected to predetermined three-dimensional stress states, ensuring that the test conditions align with actual engineering scenarios. A stepwise heating method is employed, gradually elevating the temperature to different predefined gradients and maintaining each temperature gradient for 2 hours. This approach ensures uniform temperature distribution within the culvert specimens, allowing each part to reach a state of thermal equilibrium. During the heating and constant temperature phases, precise instruments are used to monitor the linear thermal expansion along the three orthogonal directions of the culvert specimens, namely along the length, width, and height directions. Subsequently, the thermal expansion data at each temperature point are recorded, and the relationship curves between triaxial linear thermal expansion and temperature are plotted. Figure 2 presents the side crosssectional load situation of the prefabricated box culvert specimens. Analysis of these curves reveals the anisotropic and nonlinear characteristics of thermal expansion behavior. Assuming the triaxial linear thermal strain of the culvert specimens under three-dimensional stress is represented by $\gamma_{\nu}(S)$, the axial linear thermal strain by $\gamma_1(S)$, the forward linear thermal strain by $\gamma_2(S)$, the side linear thermal strain by $\gamma_3(S)$, the initial triaxial length of the culvert specimens by $M_{\nu,0}$, the triaxial linear thermal expansion at temperature S under threedimensional stress by $M_{y,S}$, and the triaxial linear thermal expansion of quartz fine sand specimens at temperature S by $M_{\nu,T}$, the calculation formula for triaxial linear thermal expansion strain under three-dimensional stress is:

$$\gamma_{\nu}(S) = \frac{M_{\nu,S} - M_{\nu,T}}{M_{\nu,0}} (\nu = 1, 2, 3)$$
(1)

The average linear thermal expansion coefficient of prefabricated box culvert specimens under three-dimensional stress is a physical quantity that describes the ratio of linear dimensional changes in different directions to temperature changes during temperature variations. In the tests, the linear

thermal expansion strain in three perpendicular directions of the specimens is precisely measured, with these strain data varying with temperature changes. Considering the practical application scenarios, three-dimensional stress, simulating real working conditions, is applied during testing, as structures in real applications are influenced not only by temperature but also by multi-directional mechanical loading. By heating the prefabricated box culvert specimens to different temperature gradients and maintaining constant temperature, a series of thermal expansion strain data are obtained. The average linear thermal expansion coefficient in each direction is determined by calculating the average of the expansion strains in different directions and combining them with the corresponding temperature changes. Assuming the triaxial linear thermal expansion strain of the culvert specimens under threedimensional stress is represented by $\gamma_{\nu}(S)$, the average triaxial linear thermal expansion strain by $\gamma_Z(S)$, and the average linear thermal expansion coefficient by $\beta_Z(S)$, with the temperature change represented by ΔS , the calculation formulas are:

$$\gamma_{Z}(S) = \frac{\gamma_{1}(S) + \gamma_{2}(S) + \gamma_{3}(S)}{3}$$
 (2)

$$\beta_Z(S) = \frac{\gamma_Z(S)}{\Delta S} \tag{3}$$



Figure 3. Stress and strain monitoring point layout diagram

Next, the testing of triaxial thermal expansion stress under three-dimensional stress in prefabricated box culvert specimens is conducted. Figure 3 gives a diagram showing the layout of stress and strain monitoring points. Initially, the culvert specimens were set up, ensuring the testing equipment can apply and maintain the required three-dimensional stress state. Pressure sensors were installed in each direction to monitor stress changes in real-time. The control system was used to gradually apply three-dimensional stress to the predetermined level, ensuring the specimen reaches a stable stress state. Once the target stress was achieved, the loading program was halted, and the position of the loading rods was fixed to form triaxial fixed boundaries. The stepwise heating process was then commenced, maintaining each temperature gradient for 2 hours to ensure uniform temperature distribution and thermal equilibrium in the specimen. Concrete triaxial stress change data during the heating and constant temperature periods were collected in real-time. Due to the temperature rise, the testing equipment also expands, affecting the results of stress measurement. Therefore, it is crucial to monitor the thermal expansion behavior of the equipment and make corresponding corrections to the measurements. After data

collection, the triaxial thermal expansion stress of the specimen under three-dimensional stress was calculated. This was done by subtracting the thermal expansion stress of a control specimen, which was not subjected to three-dimensional stress under the same conditions, to eliminate the impact of free expansion caused by the temperature rise. Assuming the triaxial thermal expansion stress of the prefabricated box culvert specimen under three-dimensional stress is represented by $\delta_v(S)$, the experimentally measured triaxial thermal expansion stress under three-dimensional stress by $\delta_{v,S}$, the triaxial stress generated by the structure under the same three-dimensional stress environment by $\delta_{v,T}$; the elastic modulus of the structure by R_T , and the experimentally measured triaxial thermal expansion strain under three-dimensional stress by $\gamma_{v,S}$, the calculation formula is:

$$\delta_{v}(S) = \delta_{v,S} - \delta_{v,T} = \delta_{v,S} - R_{T} \gamma_{v,T} (v = 1,2,3)$$
(4)

2.2 Validation of thermal expansion behavior

As a crucial component of urban infrastructure, the stability of prefabricated box culverts under thermal stress is directly related to engineering safety. To validate the applicability of theoretical models and further strengthen the research methodology combining experimental studies with theoretical analysis, this paper conducts precise testing and validation of the thermal expansion coefficient. The thermal expansion coefficients obtained experimentally allow for more accurate prediction and simulation of the thermal behavior of prefabricated box culverts in actual temperature fields and mechanical environments. This not only optimizes structural design but also confirms the applicability of theoretical models and reinforces the combined approach of experimental research and theoretical analysis.

The average linear thermal expansion coefficient of prefabricated box culvert specimens under three-dimensional stress displays an S-shaped trend with temperature changes, revealing the nonlinear characteristics of their thermal expansion behavior. In the temperature range of 100-200°C, the average linear thermal expansion coefficient shows a decreasing trend due to the evaporation of internal moisture in the concrete, leading to changes in porosity and a reduction in the overall thermal expansion capability of the material. As the temperature continues to rise, further evaporation of internal moisture causes changes in the microstructure of the material. Hence, between 200-500°C, the average linear thermal expansion coefficient of the prefabricated box culvert specimens shows a significant increase. Upon reaching 500°C, the coefficient peaks due to the decomposition of certain minerals in the concrete, resulting in considerable volumetric expansion. However, when the temperature exceeds 500°C, the mineral decomposition caused by high temperatures is nearly complete or finished, and the material begins to lose stability, leading to a gradual weakening of the thermal expansion response. As the temperature continues to increase to 600°C, the expansion coefficient declines again. Throughout the entire temperature change process, the thermal expansion behavior of the prefabricated box culvert specimens under three-dimensional stress is complex and influenced by various factors, such as material structure, internal moisture status, stress conditions, and temperature itself.

Assuming the elastic modulus is represented by R_S , the linear thermal expansion coefficient of concrete under threedimensional stress by $\beta(S)$, and the temperature change by ΔS , the following formula provides a comparison between the theoretical calculation and the experimentally obtained thermal expansion stress under three-dimensional stress:

$$\delta_z = R_S \beta(S) \Delta S \tag{5}$$

From the comparison, it is observed that under threedimensional stress conditions, the thermal expansion stress of prefabricated box culvert specimens generally increases with rising temperature, which is related to the thermal expansion characteristics of concrete material. The internal moisture of concrete gradually evaporates during heating, impacting its volume and internal pore structure, leading to changes in thermal expansion stress. Within the temperature range of 100-500°C, as moisture evaporates and chemical bonds break, the porosity within the concrete increases, gradually raising the thermal expansion coefficient and causing a slight increase in thermal expansion stress. Moreover, the rise in the thermal expansion coefficient may also be associated with the formation and expansion of microcracks within the material. As the temperature rises, the expansion of these microcracks can cause the actual thermal expansion stress to be lower than the theoretical prediction. However, in the temperature range of 500-600°C, the thermal expansion stress of the specimens shows a decreasing trend, possibly due to thermal decomposition of certain mineral components in the concrete at higher temperatures, reducing the material's stiffness. Further temperature increases may lead to the decomposition of these substances, causing a redistribution of internal stresses in the material, thus reducing the thermal expansion stress. Additionally, high temperatures may also lead to the decomposition of components like calcium carbonate in concrete, producing carbon dioxide, increasing porosity, and further lowering the material's modulus and thermal expansion stress.

In actual engineering applications, the thermal expansion behavior of prefabricated box culverts typically occurs under the influence of three-dimensional stress, significantly differing from the thermal expansion behavior under the nostress condition commonly observed in laboratory tests. Laboratory testing often overlooks the mechanical loading conditions actually endured by structures on-site, and these real stress states significantly impact the material's thermal expansion characteristics. Therefore, using the thermal expansion coefficient under no-stress conditions to predict the thermal expansion of actual structures might lead to discrepancies between calculated results and real situations, failing to accurately reflect the true performance of prefabricated box culverts in service environments. Consequently, this study compares the typical thermal expansion strain model with the thermal expansion coefficients measured under three-dimensional stress conditions. Through this comparison, the impact of threedimensional stress on thermal expansion behavior can be quantitatively assessed, further understanding the specific mechanisms by which stress conditions influence the thermal expansion coefficient.

The following formula presents the calculation for the thermal expansion strain of ordinary concrete:

$$\Delta 1/1 = 2.3 \times 10^{-11} S^3 + 9 \times 10^{-6} S$$

-1.8×10⁻⁴ (20°C ≤ S ≤ 600°C) (6)

3. TEMPERATURE-STRESS COUPLING OF PREFABRICATED BOX CULVERTS AND ITS IMPACT ON STRUCTURAL STABILITY

To accurately describe and calculate the thermal stress distribution of prefabricated box culvert specimens under varying temperature conditions and effectively assess the structure's performance under thermal loads, this paper has developed a temperature-stress coupling model based on thermoelasticity theory. By analyzing the patterns of thermal stress changes under different fire scenarios, weak points in the structure can be identified, enabling targeted reinforcement measures or design improvements to enhance the structure's fire resistance and overall stability. Figure 4 presents the schematic diagram of the temperature-stress testing system for prefabricated box culvert specimens.



Figure 4. Temperature-stress testing system for prefabricated box culvert specimens

This paper employs the principle of superposition to decompose the temperature-stress coupling analysis problem of prefabricated box culverts into two parts: external loads and temperature loads. External loads typically include gravity, traffic load, earth pressure, etc., while temperature loads involve thermal expansion or contraction due to temperature changes. This decomposition method simplifies the solution process of complex problems and helps to clearly understand the individual impacts of different types of loads on the stress response of prefabricated box culvert structures.

The paper first establishes the basic physical equations of thermoelasticity for temperature-stress coupling analysis of prefabricated box culverts. It begins by defining the displacement changes in each direction of a microelement under a three-dimensional stress state. These displacement changes, compared to the size of the microelement, yield strain components in three orthogonal directions. Based on the geometrical relationships of the microelement, linear and shear strain components in each direction can be further calculated. If the temperature changes from s_1 to s_2 , with the temperature change of the microelement being $s=s_1-s_2$, and the thermal expansion coefficient represented by β , then the length changes of da, db, dc are $(1+\beta s)da$, $(1+\beta s)db$, $(1+\beta s)dc$, respectively. The expressions for strain components are:

$$\begin{cases} \gamma_{a0} = \gamma_{b0} = \gamma_{c0} = \beta s \\ \varepsilon_{ab0} = \varepsilon_{bc0} = \varepsilon_{ca0} = 0 \end{cases}$$
(7)

Utilizing Hooke's Law, the linear and shear strains obtained in the previous step are related to the corresponding stress components. Hooke's Law provides a linear relationship between stress and strain in elastic materials, used to calculate the stress components generated inside a microelement under a given external load. Assuming the shear modulus is represented by H, the elastic modulus by R, Poisson's ratio by ω , and the volumetric stress by Φ , the calculation formulas are:

$$\begin{cases} \gamma_{a} = \frac{\partial i}{\partial a} = \frac{1}{R} [\delta_{a} - \omega (\delta_{b} + \delta_{c})] + \beta s \\ \gamma_{b} = \frac{\partial i}{\partial b} = \frac{1}{R} [\delta_{b} - \omega (\delta_{a} + \delta_{c})] + \beta s \\ \gamma_{c} = \frac{\partial i}{\partial c} = \frac{1}{R} [\delta_{c} - \omega (\delta_{b} + \delta_{a})] + \beta s \end{cases}$$

$$(8)$$

$$\mathcal{E}_{ab} = \frac{\pi_{ab}}{H}, \mathcal{E}_{bc} = \frac{\pi_{bc}}{H}, \mathcal{E}_{ca} = \frac{\pi_{ca}}{H}$$
(9)

$$H = R/2(1+\omega) \tag{10}$$

$$\boldsymbol{\Phi} = \boldsymbol{\delta}_a + \boldsymbol{\delta}_b + \boldsymbol{\delta}_c \tag{11}$$

Under the influence of temperature, the generalized Hooke's Law, which incorporates temperature effects, was applied. This law not only relates stress to mechanical strain but also includes thermal strain caused by temperature changes. Thermal strain is typically represented by the product of temperature change and the material's linear thermal expansion coefficient. Through this step, an expression of strain that includes temperature-stress coupling effects is obtained, namely:

$$\begin{cases} \delta_{a} = 2H\gamma_{a} + \frac{\omega}{1+\omega}\Phi - 2H\beta ds \\ \delta_{b} = 2H\gamma_{b} + \frac{\omega}{1+\omega}\Phi - 2H\beta s \\ \delta_{c} = 2H\gamma_{c} + \frac{\omega}{1+\omega}\Phi - 2H\beta s \end{cases}$$
(12)

$$\pi_{ab} = H\varepsilon_{ab}, \pi_{bc} = H\varepsilon_{bc}, \pi_{ca} = H\varepsilon_{ca}$$
(13)

Finally, the volumetric strain caused by temperature changes needs to be considered. Volumetric strain results from the combined effect of linear thermal expansion in three orthogonal directions. By adding up the linear thermal expansion strains in the three directions, the volumetric strain can be determined. In the temperature-stress coupling analysis of prefabricated box culverts, volumetric strain is an important parameter for assessing the overall stability of the structure under thermal loads, namely:

$$r = \gamma_a + \gamma_b + \gamma_c = \frac{1 - 2\omega}{1 + \omega} \frac{\Phi}{2H}$$

+3\beta s = \frac{1 - 2\omega}{R} \Phi + 3\beta s \text{ (14)}

Consequently, there is:

$$\Phi = \frac{R}{1 - 2\omega} (r - 3\beta s) \tag{15}$$

Assuming the thermal stress parameter is represented by α and the Lamé parameter by η , the relationship using Lamé's constants in elasticity theory is given as:

$$\begin{cases} \delta_a = 2H\gamma_a + \eta r - \alpha_s \\ \delta_b = 2H\gamma_b + \eta r - \alpha_s \\ \delta_c = 2H\gamma_c + \eta r - \alpha_s \end{cases}$$

$$\alpha = \frac{\beta R}{1 - 2\omega} = \beta(3\eta + 2H)$$

$$\eta = \frac{R\omega}{(1 + \omega)(1 - 2\omega)}$$
(16)

Further, based on continuum mechanics and thermoelastic theory, the equilibrium differential equations are derived. These equations describe how mechanical balance and energy balance within materials are expressed at a microscopic level through the interrelationships between stress, strain, and temperature. These equations combine Newton's second law and Hooke's law to describe the stress distribution inside a microelement. Assuming the components of body force on each axis are represented by A, B, and C, the equations are:

$$\begin{cases} \frac{\partial \delta_{a}}{\partial a} + \frac{\partial \pi_{ba}}{\partial b} + \frac{\partial \pi_{ca}}{\partial c} + A = 0\\ \frac{\partial \delta_{b}}{\partial b} + \frac{\partial \pi_{cb}}{\partial c} + \frac{\partial \pi_{ab}}{\partial a} + B = 0\\ \frac{\partial \delta_{c}}{\partial c} + \frac{\partial \pi_{ac}}{\partial a} + \frac{\partial \pi_{bc}}{\partial a} + C = 0 \end{cases}$$
(17)

By substituting the above constitutive relations into the equilibrium differential equations, the displacement components of the prefabricated box culvert structure under specific temperature and stress conditions can be solved. These displacement components typically appear in the form of partial differential equations and can be solved using numerical methods such as finite element analysis. The expressions for displacement components directly link the structure's deformation with internal stress, external load, and temperature change, providing an important tool for assessing the behavior of structures under actual operating conditions. The expressions are as follows:

$$\begin{cases} (\eta + H)\frac{\partial\gamma}{\partial a} + H\nabla^{2}i - \alpha \frac{\partial s}{\partial a} + A = 0\\ (\eta + H)\frac{\partial\gamma}{\partial b} + H\nabla^{2}i - \alpha \frac{\partial s}{\partial b} + B = 0\\ (\eta + H)\frac{\partial\gamma}{\partial c} + H\nabla^{2}i - \alpha \frac{\partial s}{\partial c} + C = 0 \end{cases}$$
(18)

$$\nabla^2 = \frac{\partial^2}{\partial a^2} + \frac{\partial^2}{\partial b^2} + \frac{\partial^2}{\partial c^2}$$
(19)

Thermoelastic geometric equations can clearly describe the deformation of box culvert structures under the influence of temperature changes and stress. To construct these geometric equations, the basic relationship between strain and displacement within the framework of thermoelasticity must first be established. This usually begins with the definition of strain components, i.e., strain is a function of the gradient of the displacement field. For prefabricated box culverts, this means considering the displacement of each point of the culvert under the influence of temperature and external loads. Subsequently, to account for temperature effects, the thermal expansion coefficient and how the temperature field affects the displacement of material points are introduced. The product of the thermal expansion coefficient and temperature change represents the thermal strain component caused by temperature change, which must be combined with the strain component caused by mechanical forces to obtain the total strain component. Finally, the strain-displacement relationship and compatibility conditions are combined to form a complete geometric equation, expressed as follows:

$$\{\gamma\} = \{\gamma_a, \gamma_b, \gamma_c, \varepsilon_{ab}, \varepsilon_{bc}, \varepsilon_{ca}\}^{S}$$
$$= \left\{\frac{\partial i}{\partial a}, \frac{\partial i}{\partial b}, \frac{\partial i}{\partial b}, \frac{\partial n}{\partial a} + \frac{\partial i}{\partial b}, \frac{\partial q}{\partial b} + \frac{\partial n}{\partial c}, \frac{\partial i}{\partial c} + \frac{\partial q}{\partial a}\right\}^{S}$$
(20)

To ensure the continuity and consistency of internal displacement distribution in box culvert structures under the combined action of temperature changes and external loads, this study constructs the displacement coordination equations of thermoelasticity. Initially, considering thermal strains caused by temperature, displacement vectors are taken into account, typically achieved by combining temperature gradients with the material's thermal expansion coefficient. Thermal strain directly contributes to displacement components and thus must be reflected in the displacement coordination equations. This combines thermal strain with elastic strain caused by external loads on the prefabricated box culverts. To ensure that the material does not undergo physically impossible deformations, compatibility conditions are used to establish coordinated relationships between various displacement components. These conditions ensure the continuity of the displacement field, allowing the displacement vector to naturally transition along different directions. Integrating all the above considerations forms the displacement coordination equation, a set of partial differential equations that describe the interrelationships between displacement components. These equations often involve second-order derivatives of displacement to ensure the continuity of strain, expressed as:

$$\begin{cases}
\frac{\partial^{2} \gamma_{a}}{\partial b^{2}} + \frac{\partial^{2} \gamma_{b}}{\partial a^{2}} = \frac{\partial^{2} \varepsilon_{ab}}{\partial a \partial b} \\
\frac{\partial^{2} \gamma_{b}}{\partial c^{2}} + \frac{\partial^{2} \gamma_{z}}{\partial b^{2}} = \frac{\partial^{2} \varepsilon_{bc}}{\partial b \partial c} \\
\frac{\partial^{2} \gamma_{z}}{\partial c^{2}} + \frac{\partial^{2} \gamma_{x}}{\partial b^{2}} = \frac{\partial^{2} \varepsilon_{ca}}{\partial c \partial a} \\
\frac{\partial}{\partial z} \left(\frac{\partial \varepsilon_{ca}}{\partial b} + \frac{\partial \varepsilon_{ab}}{\partial c} + \frac{\partial \varepsilon_{bc}}{\partial a} \right) = 2 \frac{\partial^{2} \gamma_{a}}{\partial b \partial c} \\
\frac{\partial}{\partial b} \left(\frac{\partial \varepsilon_{ab}}{\partial c} + \frac{\partial \varepsilon_{bc}}{\partial a} + \frac{\partial \varepsilon_{ca}}{\partial b} \right) = 2 \frac{\partial^{2} \gamma_{b}}{\partial c \partial a} \\
\frac{\partial}{\partial c} \left(\frac{\partial \varepsilon_{bc}}{\partial a} + \frac{\partial \varepsilon_{ca}}{\partial b} + \frac{\partial \varepsilon_{ab}}{\partial c} \right) = 2 \frac{\partial^{2} \gamma_{b}}{\partial c \partial a}
\end{cases}$$
(21)

The solution of the temperature-stress coupling model for prefabricated box culverts involves setting a series of physically reasonable boundary conditions to simulate the behavior of culverts in actual engineering. These boundary conditions typically include: 1) Mechanical boundary conditions: These define the stress or displacement distribution of the culverts under the action of external loads. 2) Thermal boundary conditions: These describe the behavior of culverts in thermal environments, such as exposure to different temperatures or fire impact. 3) Coupled boundary conditions: Considering temperature-stress coupling effects, boundary conditions that involve both temperature and stress impacts are needed. 4) Initial conditions: These significantly impact the solution process, especially in dynamic analysis or simulating the time response of culverts. Assuming the surface stress components of the boundary microelement are represented by \overline{A} , \overline{B} , \overline{C} and the normal direction cosines are 1, *l*, v, the expressions for stress boundary conditions are:

$$\begin{cases} \overline{A} = \delta_a m + \pi_{ba} l + \pi_{ca} v \\ \overline{B} = \delta_b l + \pi_{cb} v + \pi_{ab} m \\ \overline{C} = \delta_z v + \pi_{ac} m + \pi_{bc} l \end{cases}$$
(22)

Finite element analysis is used to solve the coupled model, obtaining the culvert structure response under various temperature and stress conditions, including temperature distribution, displacement field, and stress field. Particularly, attention is paid to the locations of maximum stress and strain in the structure and the trends of these values with temperature changes. Next, it's analyzed whether these stresses and strains exceed the permissible limits of the material, especially considering material performance degradation at high temperatures. Additionally, the expansion or contraction caused by temperature leading to functional defects, such as crack opening or closure or failure of connectors, is evaluated.

Combining the actual usage conditions of prefabricated box culverts, such as load history, construction quality, and material aging, the potential performance of the structure in real working environments is assessed. For extreme events like dynamic loading or fire, the structure's stability under rapid temperature changes in a short period is specifically considered. Further, through sensitivity analysis and parameter studies of the solution results, key factors affecting the stability of box culvert structures, such as support conditions, material thermal expansion coefficients, and thermal conductivity, need to be identified. With this information, potential issues under different working conditions can be predicted, and mitigation measures can be formulated.

Finally, based on the solution results, the safety margin of the structure can be assessed, design optimization can be carried out, or recommendations for reinforcement or repair can be made. If the model predicts stability risks under certain conditions, the structural safety performance can be improved by adjusting design parameters or changing construction processes.

4. EXPERIMENTAL RESULTS AND ANALYSIS

Figure 5(a) presents the thermal expansion rate data of prefabricated box culvert specimens at different temperatures along three directions: a, b, and c, including both heating and cooling processes. It is observed that the thermal expansion

rate generally increases with temperature, as higher temperatures typically cause an increase in the vibration amplitude of internal particles, leading to an expansion of the material's volume. The thermal expansion and contraction behaviors of the material exhibit asymmetry, meaning the trends of thermal expansion rate during heating and cooling are different. This asymmetry is due to reasons such as reorganization of the material's internal microstructure or phase transitions. The figure also shows that the thermal expansion rate varies in different directions, indicating the anisotropic properties of the box culvert specimens. Such anisotropy is caused by the composition of the material, processing methods, or the directional nature of the internal structure.



(b) Different positions



Figure 5(b) shows the data of the linear thermal expansion rate at the top and bottom of the prefabricated box culvert at different temperatures, including both heating and cooling processes. The figure indicates that the thermal expansion rates at both the top and bottom of the box culvert first increase to a peak value during heating and then decrease with further temperature rise, even becoming negative. This phenomenon suggests that the material expands initially with temperature increase but later undergoes thermal contraction or damage due to thermal stress, fatigue, or other mechanisms, such as thermal cracking, aging, or other thermally induced damages. The inconsistency of thermal expansion rates between heating and cooling processes represents a thermal hysteresis effect. This effect is due to irreversible changes in the physical properties of the material during heating and cooling, or adjustments of internal stresses during temperature changes, causing different behaviors during heating and cooling. The differences in thermal expansion rates between the top and bottom reflect the variations in material properties or stress states at different parts of the box culvert. The top is usually more susceptible to external environmental factors like temperature changes, while the bottom, due to constraints from the foundation or internal structure, exhibits different thermal expansion behaviors. Particularly noteworthy is that at temperatures above 800°C, the thermal expansion rates at both the top and bottom of the box culvert significantly decrease, reaching notable negative values. This implies structural changes in the material at high temperatures, such as phase transitions, melting, cracking, or other forms of damage, leading to a reduction in overall volume.



Figure 6. Thermal expansion coefficients of prefabricated box culvert specimens under three-dimensional stress state

The data on thermal expansion coefficients with temperature changes presented in Figure 6 reflects the physical behavior of prefabricated box culvert materials under threedimensional stress conditions. The trend of the thermal expansion coefficient with temperature change is not monotonically increasing or decreasing, indicating that the material's thermal behavior is influenced by various factors, such as phase transitions, chemical changes, release of internal stresses, or temperature-dependent structural changes in the material. Between 100°C and 200°C, the thermal expansion coefficient significantly decreases from 9.75 to 8.065, due to some form of structural reorganization of the material in this temperature range, leading to a weakened micro-level thermal expansion capability. From 200°C to 500°C, the thermal expansion coefficient generally increases from 8.065 to 12.833, typically associated with an increase in the kinetic energy of internal particles, where particle vibrations increase with rising temperature, causing material expansion. Between 500°C and 600°C, there is a slight decrease in the thermal expansion coefficient from 12.833 to 11.945, indicating that parts of the material start to lose structural integrity at high temperatures, or the material approaches or undergoes some form of phase transition. It can be concluded that the relationship between the thermal expansion coefficient and temperature of prefabricated box culvert specimens is complex, reflecting changes in the material's microstructure and macroscopic stress state with temperature. This nonlinear behavior suggests that in engineering applications, the thermal expansion coefficient should not be assumed constant in

thermal environments. Instead, it should be considered as a function of temperature to ensure accurate prediction and understanding of the thermal behavior of the culverts across the entire temperature range.



Figure 7. Comparison of thermal expansion coefficients of prefabricated box culvert specimens under different states

Figure 7 shows a comparison of the thermal expansion coefficients of prefabricated box culvert specimens under three-dimensional stress conditions with those under certain standard specifications. To analyze these data and draw conclusions, we can focus on the differences between the two sets of data and their trends at different temperatures. In the low-temperature range (0°C to 200°C), the thermal expansion coefficient under three-dimensional stress conditions decreases from 11 to 8, while the standard specification's coefficient increases from 9 to 10. This indicates that at low temperatures, the experimental values start higher than the standard values but gradually decrease and eventually fall below the standard values. This suggests that in practical applications, the culvert material expands more at low temperatures than the materials suggested by standard specifications, but as the temperature increases, its expansion performance becomes inferior to that of the standard materials. In the mid-temperature range (200°C to 500°C), the thermal expansion coefficient under three-dimensional stress conditions first increases, then surpasses the standard specification value, reaching a maximum of 13 at 500°C, while the standard specification value at this temperature is 15.5. This indicates that in the mid-temperature range, the experimental material's thermal expansion coefficient is lower than the standard specification values, showing less sensitivity to temperature in the experimental conditions than the materials assumed in the standard specifications. In the hightemperature range (500°C to 600°C), the thermal expansion coefficient under three-dimensional stress conditions peaks at 500°C and then slightly decreases to 12.5, while the standard specification value continues to increase to 18. This shows that as the temperature further increases, the performance of the material under experimental conditions in terms of thermal expansion is less than that of the standard specification recommended materials. Overall, the thermal expansion coefficients measured under three-dimensional stress conditions in the experiments are lower across the entire temperature range than those of the standard specifications, especially more pronounced in the mid to high-temperature range.



Figure 8. Thermal expansion stress curves of prefabricated box culvert specimens under three-dimensional stress state

Analyzing the data in Figure 8 reveals that the two specimens exhibit different patterns of change in thermal expansion stress over time in *a*-direction and *b*-direction. The data shows that the thermal expansion stress of both specimens reflects the complex behavior of prefabricated box culvert materials under a three-dimensional stress state. In engineering design and analysis, it is important to consider this type of non-linearity and directional dependency. Specifically, the stress decrease phenomenon in the a-direction for Specimen 2 requires further research to determine its cause, and this behavior should be considered in design implications. Meanwhile, the trend towards stability in the b-direction indicates that the material or structural design in this direction is more robust. This information is very useful for predicting and enhancing the performance of prefabricated box culverts in thermal environments.

Figure 9 displays a comparison between the theoretical values of thermal expansion stress and multiple experimental values of prefabricated box culvert specimens at different temperatures. The data indicates that both theoretical and experimental values show a general increasing trend with rising temperature, yet there are discrepancies in specific values. At lower temperatures, experimental values tend to be lower than theoretical values, which could be due to initial defects in the materials under real conditions or limitations in the experimental setup, such as uneven thermal conduction or issues with the precision of testing equipment. As the temperature increases, experimental values in some cases exceed theoretical values, while in other cases, they remain

lower. This suggests that the performance of the specimens in actual tests is influenced by various factors, including changes in the micro-structure of materials, unevenness in the thermal environment, or boundary conditions during testing. The analysis indicates that prefabricated box culvert specimens under three-dimensional stress conditions exhibit a certain degree of variability in thermal expansion stress. The differences between actual test values and theoretical values are due to experimental conditions, specimen preparation processes, precision of testing equipment, material inhomogeneity, or other unknown factors. In engineering applications, this variability needs to be fully considered, especially under high-temperature conditions, where materials may exhibit behaviors beyond standard theoretical predictions.

Table 1 presents the thermal expansion stress of two specimens in three directions (a, b, c directions) as a function of temperature. For both specimens, the thermal expansion stress in all directions generally shows an increasing trend with rising temperature. This is consistent with the basic theory of thermal expansion, which states that a body expands when heated, thereby increasing internal stress. For Specimen 1, the *a*-direction exhibits the highest thermal expansion stress at 600°C (85.234), while the *c*-direction shows the lowest at the same temperature (57.412). The *b*-direction reaches its peak thermal expansion stress at 500°C (52.364) but slightly decreases at 600°C. This suggests that under high-temperature conditions, the material's thermal stability is lower along the *a*-direction, or the constraints in the *a*-direction lead to more accumulation of thermal stress. For Specimen 2, the thermal expansion stress in the *a*-direction (88.249) is also highest at 600° C, but compared to Specimen 1, the *c*-direction shows a decrease in thermal expansion stress at 600° C (48.237), indicating differences in material uniformity between specimens or variations in testing conditions. It can be concluded that for both specimens, the thermal expansion stress under three-dimensional stress conditions increases with rising temperature. However, the degree of increase and the final values vary in the three directions, reflecting the anisotropy of the material in different directions.



Figure 9. Comparison of theoretical and experimental values of thermal expansion stress of prefabricated box culvert specimens under three-dimensional stress state

Table 1. Thermal expansion stress of prefabricated box culvert specimens under three-dimensional stress conditions



Figure 10. Comparison of crack fracture rates in specimens with high and low thermal expansion coefficients

Figure 10 lists the fracture rates of corner cracks, and continuous and discontinuous cracks for prefabricated box culvert specimens of different ages under conditions of varying thermal expansion coefficients. As shown in Figure 10-a, specimens under high thermal expansion coefficient conditions are more prone to corner crack fractures, especially

as they age, with a significant increase in fracture rates. This aspect should be considered in design and material selection. Under low thermal expansion coefficient conditions, the fracture rates are lower and the trend of increase with age is not as pronounced, becoming significant only in older culverts (over 30 years). In engineering design and maintenance strategies, particular attention should be paid to culverts in the middle or later stages of their lifespan and with high thermal expansion coefficients, as these structures are more prone to cracking. Frequent inspections, maintenance, and reinforcement measures are necessary for culverts with higher thermal expansion coefficients to ensure their structural safety throughout the design life. Figure 10-b shows that under high thermal expansion coefficient conditions, the fracture rates generally increase with the age of the culvert. This trend is particularly noticeable under conditions >6.0, indicating that higher thermal expansion coefficients lead to greater accumulation of thermal stress, increasing the propensity for crack development. The highest fracture rates occur in prefabricated box culverts at 30 and 40 years, due to material degradation from long-term thermal cycling or internal stresses reaching a critical point where continuous cracks form. Under low thermal expansion coefficient conditions, fracture rates are relatively lower, especially in younger culverts (5 to 15 years). However, as the culvert ages, fracture rates also show an increasing trend, as even lower thermal expansion can accumulate over time, causing damage. A sudden increase in fracture rates at 25 years under low thermal expansion

coefficient conditions emphasizes that even with low thermal expansion coefficients, thermal stress can exceed the material's durability over time.

The fracture rate data indicate that the thermal expansion behavior of box culvert specimens significantly impacts their structural stability. As prefabricated box culverts age, the risk of crack development increases due to material degradation and the cumulative effect of thermal stress, even under low thermal expansion coefficient conditions. High thermal expansion coefficients exacerbate this issue, making cracks more likely to develop earlier and with higher fracture rates, especially as the culverts reach middle to later ages. In engineering design and maintenance, these findings underscore the importance of considering thermal expansion coefficients in predicting and enhancing the stability of box culvert structures. Suitable materials should be selected, and preventive measures should be taken during design to minimize stress caused by thermal expansion. Proper inspections and maintenance throughout the structure's lifecycle are crucial for monitoring crack development and implementing timely repairs.

Table 2. Test results of internal damage in prefabricated box culvert specimens

Specimen Number		Pre-Experiment Ultrasonic Velocity	Post-Experiment Ultrasonic Velocity	Degree of Damage	Average Degree of Damage
Specimen 1	Measure point 1	6.895	6.582	0.04	
	Measure point 2	4.897	2.789	0.42	
	Measure point 3	4.489	2.124	0.53	0.39
	Measure point 4	3.885	1.895	0.51	
	Measure point 5	3.562	1.783	0.48	
	Measure point 6	3.612	2.236	0.42	
	Measure point 7	4.325	2.843	0.32	
	Measure point 8	4.692	3.561	0.22	
	Measure point 9	3.785	2.134	0.42	
Specimen 2	Measure point 1	6.395	5.124	0.2	0.19
	Measure point 2	4.125	4.236	0.03	
	Measure point 3	4.128	3.568	0.11	
	Measure point 4	3.692	2.568	0.29	
	Measure point 5	3.369	3.366	0.01	
	Measure point 6	4.125	4.127	0.04	
	Measure point 7	4.023	2.689	0.33	
	Measure point 8	4.625	3.265	0.28	
	Measure point 9	3.369	2.625	0.22	

Table 2 provides the test results of ultrasonic velocities before and after the experiment for two prefabricated box culvert specimens, along with their calculated degrees of damage. The table shows that Specimen 1 has a significantly higher average degree of damage compared to Specimen 2. This suggests that Specimen 1 experienced more severe damage during manufacturing, usage, or testing, or there were inherent defects in the original materials. The thermal

expansion behavior of prefabricated box culverts leads to internal damage, particularly if the material has a high thermal expansion coefficient or the thermal expansion is uneven, which can generate substantial thermal stresses within the material, exacerbating damage and the formation of cracks. The higher degree of damage in Specimen 1 indicates that if this specimen represents a particular type of box culvert material or structure, then this type of culvert requires extra attention to ensure its long-term structural stability. It may necessitate more frequent monitoring and maintenance, or improvements in design and material selection to reduce the impact of thermal stresses on the structure. From an engineering application perspective, these test results underscore the importance of considering thermal expansion behavior in the design and material selection of prefabricated box culverts, as well as the necessity of regular assessment and monitoring of the structural health of culverts. Appropriate design and maintenance measures can help extend the lifespan of the culverts and ensure their safe operation.

5. CONCLUSION

This study focuses on systematically testing and validating the thermal expansion behavior of prefabricated box culverts and its impact on structural stability. Through experimental measurements, the study evaluated the triaxial linear thermal expansion strain and the average linear thermal expansion coefficient of prefabricated box culverts under threedimensional stress conditions. These experiments were conducted by testing culvert specimens under various temperature conditions, revealing the thermal expansion behavior of the culvert materials in three principal directions. The comparison of experimental results with theoretical predictions validated the accuracy of the thermal expansion coefficients under three-dimensional stress conditions. Observed differences between thermal expansion stress in experiments and theoretical models are attributed to material inhomogeneity, testing conditions, or the complexity of boundary conditions in the experiments. The study developed a temperature-stress coupling model, simulating the variation of thermal stress in prefabricated box culverts under different temperature conditions. Analysis of this model revealed the potential impact of temperature changes on the structural stability of culverts, especially in long-term engineering applications. The in-depth analysis of the thermal stress variation patterns under different temperature conditions and their impact on structural stability shows that the magnitude and variation pattern of thermal expansion coefficients are crucial for understanding and predicting structural behavior.

This research provides new theoretical foundations and methodologies for engineering design and safety assessment. Accurate prediction of thermal expansion behavior allows for the optimization of culvert design, preventing structural damage due to temperature changes, thereby enhancing the safety and durability of the structure. The importance of considering thermal expansion behavior in the design and assessment process of prefabricated box culverts is emphasized in this study. Thermal expansion stress significantly affects structural stability, particularly under extreme temperature conditions or long-term cyclic loading. The combination of experimental results and theoretical analysis offers tools for predicting and managing these thermal stresses, which is vital for ensuring the long-term stability and safety of culvert structures. In engineering practice, these findings can guide more rational design choices and maintenance strategies, ensuring the structure remains functional and safe throughout its expected lifespan.

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