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Optimal Design of Plate-Fin Heat Sink under Natural Convection Using a Particle Swarm Optimization Algorithm

C. H. Liang [1, 3*], S. Zeng [1], Z. X. Li [2], D. G. Yang [1] and S. A. Sherif [3]

[*1] School of Mechano-Electronic Engineering, Guilin University of Electronic Technology,
Guilin, 541004, China
[2] Department of Environmental Engineering, Shan Xi University, Taiyuan, 030006, China
[3] Department of Mechanical and Aerospace Engineering, University of Florida, Gainesville,
32601, USA

Email: lianghang@guet.edu.cn

ABSTRACT

The purpose of this study is to find the optimal designing parameters of a plate-fin heat sink under natural convection using the Particle Swarm Optimization (PSO) Algorithm. Minimization of entropy generation rate under given space restrictions is considered as objective functions. All relevant design parameters for plate-fin heat sinks are the fin height, fin number, fin thickness. The constraints of the variables are set according to the suggestion structure design. And this three variables influence on entropy generation are presented. In the present study, In order to prevent the size of the heat sink is too large, we use the penalty function method in this study. Then the code for the PSO is written in MATLAB. On this basis, the optimal size of heat sink was obtained through the particle swarm algorithm for numerical simulation of this model: fin height is 44.8mm, number of fins is 25, fin thickness is 0.6mm and base temperature is 342.6241k.

Keywords: Pate-fin, Heat sink, Particle swarm optimization, Entropy generation, Optimization.

1. INTRODUCTION

Since the recent development in integrated circuit (IC) technology over the past few decades, electronics have become faster，smaller and more powerful, which leads to an ever-increasing heat generation rate from electronics devices. This trend inevitably leads to the increase of heat generation rate per volume of the electronics devices [1, 2]. If the heat can not be removed timely, the components will accelerate the ageing. Even worse, it will affect the normal operation of the electronic device [3]. Therefore, thermal management becomes a fundamental but crucial element in electronic product design. Many cooling methods have been proposed to maintain the temperature of electronic components in safety zone [4-6], like thermoelectric cooling, air cooling and liquid cooling. Due to their inherent simplicity, operational safety and low long-term cost, natural convection heat sinks have been widely used in cooling electronic components. The factors that affect the performance of a heat sink are the thermal conduction resistance, choice of material, protrusion design and surface treatment. Also, heat sink attachment methods affect the die temperature of the electronic components. The massive parameters make the optimal design of the heat sinks a challenge.

There is an increasing interest among researchers in the development of heat sink processes for heat dissipation and many optimization process for heat sink design have been proposed. Many researchers [7-9] have conducted experiments to study the performance of heat sinks. For optimal design of heat sink, the method of entropy generation minimization introduced by Bejan [10,11], provides a procedure for simultaneously optimization of heat sink design parameters as they relate to not only viscous effects but also thermal performance. Using the entropy generation minimization technique, Culham and Muzychka [12] optimized a plate-fin heat sink equipped with a flow-through air inlet system. Further, by considering the geometry constraint effect on the performance of heat sink, Shih and Liu [13] concluded that the optimal designed heat sink process outperforms the conventional ones. Iyengar and Bar-Cohen [14] presented a coefficient of performance analysis for plate fin heat sinks in forced convection. In their study, a viable technique was provided for combining least-material optimization with the entropy minimization methodology.

In order to improve the thermal performance of the heat sink about the CPU, Chen et al [15] apply a finite element method to investigate the heat transfer phenomena of a heat sink process firstly, then, a real coded genetic algorithm was applied to search for an optimal set of plate-fin parameters. The optimal geometry size of heat sink was obtained by Adewumi et al [16] using the computational fluid dynamics code with a goal-driven optimization algorithm. Waghmare et al [17] used the teaching-learning-based optimization algorithm for optimization of plate-fin heat sink equipped with flow-through and impingement-flow air cooling system. Height of fins, number of fins, spacing between two fins and oncoming air velocity are considered as the design variables. Finally, the author concluded that the plate-fin heat sink with flow-through air cooling system is better than the plate-fin heat sink with impingement-flow cooling system. Faraji et al [18] apply the TDMA algorithm to optimize the thermal performance of a phase change material heat sink and develop A mathematical model about it. The results show that the optimized heat sink has good performance. For the pin-fin heat sink, Chiang et al. [19] optimized the design parameters of Pin-Fin Heat Sink with multiple thermal characteristics using the grey-fuzzy logic based on the orthogonal arrays. In addition, the effect of the design parameters on the thermal performance characteristics of the heat sink was found using the analysis of variance (ANOVA).

However, there has been less research work on using Particle swarm optimization (PSO) for heat sink design. Since the particle swarm algorithm is put forward by Kennedy and Eberhart [20], the successful use of the Particle swarm optimization (PSO) for practical problems has been reported every year. The goal of this paper is to demonstrate the geometrical design of a plant-fin heat sink under natural convection by using PSO. The objective functions known as entropy generation rate with five constraints have been taken to measure the performance of the heat sink. Three optimization variables are fin number, fin height, fin thickness respectively. According to our design experience, we have found that an optimal design simply based on a lowest entropy generation rate often leads to a larger size of heat sink. So, in order to meet customer needs, the penalty function is used in this study to prevent the size of the heat sink is too large.

This paper is organized in the following manner. First, we present the object function in section 2. The following section then briefly describes the particle swarm algorithm. In order to meet the size requirements of the heat sink, the constraint condition is given in section 4. Section 5 shows the flowchart of PSO algorithm. The next section is to analyze the results and verify the accuracy of the results. Finally, concluding remarks are given in Section 7.

2. MATHEMATICAL MODEL

For optimization, an overall maximum volume of 290mm × 84mm × 75mm heat sink model as shown in Figure 1 has been taken to optimize based on Particle Swarm Optimization Algorithm. We assumed that a total heat dissipation of 28.8 W is uniformly applied over the base plate of the heat sink. The thickness of the base plate of the heat sink, t_b, is 4 mm. The thermal conductivity of the heat sink material, K_a, is set to be 200w/mk and the ambient temperature around heat sink,T_2 , is set to be 303.15k.

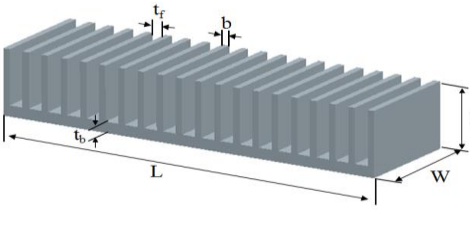

Figure 1. The geometric parameters of the plate fin heat sink

According to the methodology of Bejan [11], entropy generation rate ($\dot{S}_{gen}$) for extended surface under free convection is defined by the following relationship

$$\dot{S}_{\mathrm{gen}} = \frac{Q\theta_{\mathrm{b}}}{T_2^{\ 2}} \tag{1}$$

where T_2 is ambient temperature, Q , and θ_{b} are total heat dissipated from heat sink and the temperature excess of the heat sink base plate respectively. The relationship between the temperature excess of the heat sink (θ_{b}) and the overall heat sink resistance is defined as

$$\theta_{\mathrm{b}} = QR_{\mathrm{sink}} \tag{2}$$

Therefore, entropy generation rate is rewritten as

$$\dot{S}_{\mathrm{gen}} = \frac{Q^2 R_{\mathrm{sink}}}{T_2^{\ 2}} \tag{3}$$

The overall thermal resistance of the heat sink, R_{sink} , is defined as

$$R_{\mathrm{sink}} = R_{\mathrm{total}} + R_{\mathrm{base}} \tag{4}$$

R_{total} is the total thermal resistance that is resulted from the fins and the exposed base plate and is given by

$$R_{\mathrm{total}} = \frac{1}{\dfrac{n}{R_{\mathrm{fin}}} + hbL(n-1)} \tag{5}$$

where n is fin number, h is heat transfer coefficient, b, L are fin space and fin length respectively. And R_{fin} is the thermal resistance of a single fin. It will be modeled using the solution for a straight fin with an adiabatic tip.

$$R_{\mathrm{sink}} = \frac{1}{\sqrt{hPK_{\mathrm{a}}A_{\mathrm{c}}}\tanh(mH)} \tag{6}$$

where

$$m = \sqrt{\frac{hP}{K_{\mathrm{a}}A_{\mathrm{c}}}} \tag{7}$$

$$A_{\mathrm{c}} = Lt \tag{8}$$

$$P = 2(L+t) \tag{9}$$

And H is the height of fin, t is fin thickness of heat sink, K_{a} is the thermal conductivity of air.

Besides, the bulk of heat sink material's thermal resistance, R_{base} , is given by

$$R_{\mathrm{base}} = \frac{t_{\mathrm{b}}}{KLW} \tag{10}$$

where t_b is the fins thickness of base plate, K is the thermal conductivity of heat sink. L, W are fin length and heat sink width, respectively.

The heat transfer coefficient of the plant-fin heat sink under natural convection is given by [21]:

$$h = \frac{K_a}{b}\left[\frac{576}{EI^2} + \frac{2.873}{EI^{0.5}}\right]^{-0.5} \tag{11}$$

Here, EI is the Elenbaas number defined as

$$EI = \frac{\rho^2 \beta g C_p b^4 \bar{\theta}}{v_f K_a L} \tag{12}$$

$\bar{\theta}$ is the average temperature difference between the heat sink and the ambient air, defined as

$$\bar{\theta} = T_1 - T_2 \tag{13}$$

And fin efficiency

$$\eta = \frac{\tanh(mH)}{mH} \tag{14}$$

To perform the thermal analysis for the plate-fin heat sink under free convection, the entropy generation rate as the objective function can be defined as follows

$$\dot{S}_{gen} = f(n,t,H) = f(x_i) \tag{15}$$

where n is the number of fins, t is the thickness of fin, H is the height of the fin in meter, T1 is base temperature of the heat sink, and the x_i is the optimized variables in the object function.

In this study, the assumptions for the analysis as follows:

(1) No spreading or constriction resistance.

(2) Constant material thermo-physical of both air and solid.

(3) Adiabatic fin tips.

(4) Uniform heat flux through entire base plate bottom surface.

3. PARITICLE SWARM OPTIMIZATION (PSO)

In 1995, Particle Swarm Optimization (PSO) developed by Kennedy and Eberhart [20] is an evolutionary computation technique for solving global optimization problems. It was inspired by the choreography of bird flocking and fish schooling. PSO is a simple algorithm as well as fast, high-quality and effective. So it can be used in a wide variety of optimization problems. Each potential solution, called a particle, and each of them flies in the N-dimensional problem space with a velocity. In the simulation, each particle moves toward the optimum point based on its present velocity, its previous experience and the experience of its neighbors. The updates of the particles are calculated using the following equations.

$$\begin{aligned} v(i,:) &= wv(i,:) + c_1 rand_1(pbx(i,:) - px(i,:)) \\ &+ c_2 rand_2(gbx - px(i,:)) \end{aligned} \tag{16}$$

$$px(i,:) = px(i,:) + v(i,:) \tag{17}$$

where i=1,2,…; i is the particle index; pbx denotes the best previous position that the corresponding particle has achieved; gbx represents the global best location; c1 and c2 are the acceleration factor, and ‘c1’ as cognitive parameter represents the confidence the particle has in itself and ‘c2’ as social parameter represents the confidence the particle has in swarm; rand1 and rand2 are random numbers with a range of [0, 1]; New position of particle is calculated by Eq.(17). In order to improve the convergence performance of PSO, Shi and Eberhart discussed the setting of inertia factor in several articles [22]. At present, the most commonly used is the linear decreasing weight (LDW) strategy proposed by Shi. The inertia weight ‘w’ is given by

$$w = w_{max} - iter(w_{max} - w_{min}) / iter_{max} \tag{18}$$

where *iter* is the current number of iterations and $iter_{max}$ is the maximum number of iterations

4. THE COMSTRAINT CONDITIONST

The objective of function is the minimization of entropy generation rate considering linear and nonlinear inequality constraints as follow

$$\min \dot{S}_{gen} = f(n,t,H) = f(x_1,x_2,x_3) \tag{19}$$

s.t.

$g_1 : (L - nt)/(n-1) - 0.014 \le 0$

$g_2 : H(n-1)/(L-nt) - 14 \le 0$

$g_3 : 1 - H(n-1)/(L-nt) \le 0$

$g_4 : 20 \le n \le 28$

$g_5 : 0.0006 \le t \le 0.0015$

$g_6 : 0.035 \le H \le 0.045$

In the above constraints, g1 indicate that the fin gap should less than 1.4mm. And two constraints about g2 and g3 indicate that the ratio of the height and thickness of the fins should lie in the range between 1 and 14 due to limited space for installation. In addition to the above constraints, fin number as g4 showed should lie in the range between 20 and 28, fin thickness as g5 showed should lie in the range between 0.6mm and 1.5mm, fin height should lie in the range between 35mm and 45mm. Equation (15) without considering constraint conditions, the right fitness function as shown below:

$$fitness = \begin{cases} \dot{S}_{gen} \\ \dot{S}_{gen} + \text{constant} \end{cases} \tag{20}$$

where ‘constant’ is set to 1000 as the penalty constant. The entropy production rate is equal to the fitness function when a solution meets the constraint about the structure size of the plate-fin heat sink. When a solution does not satisfy the restriction condition, the fitness function is equal to the sum of entropy production rate and the penalty constant.

And the velocity of the particle on each dimension are confined to a maximum velocity $v_{i\max}$ as the Eq. (21) and (22) showed

$$v_i \le v_{i\max} \tag{21}$$

$$v_{i\max} = c_3 (px_{i\max} - px_{i\min}) \tag{22}$$

where $v_{i\max}$ is the maximum allowed velocity of a particle in ith dimension. $px_{i\min}$ and $px_{i\max}$ are the minimum and maximum positions of the particles in ith dimension respectively. There, the value of c_3 is 0.2.

5. MAIN ALGORITHM

The particle's best results and overall best solution were obtained using particle swarm optimizer by change each particle's velocity and position. The flowchart of PSO algorithm is shown in Figure 2.

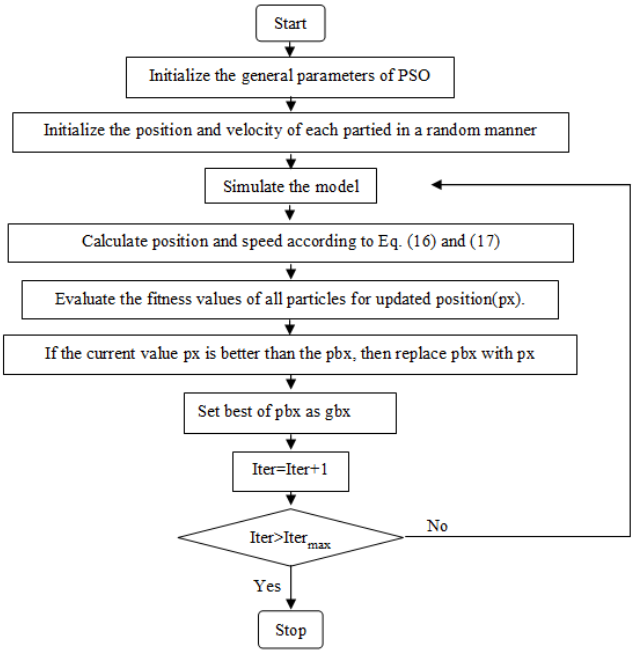

Figure 2. The flowchart of PSO algorithm

6. THE RESULTS AND ANALYSIS

The code for the PSO is written in MATLAB. After obtaining the optimum solution in the PSO approach, PSO algorithm is run 200 times by considering the following parameters: Number of particles (n) is 50; the acceleration factor c1=c2=2; the maximum velocity ($v_{i\max}$) of each dimension as shown in equation (21); the maximum number of generations ($iter_{\max}$) is 200; the inertial weight (w) lie in the range between 0.4 and 0.9; Results are shown in Figure 3. After 200 iterations, it is interesting to observe that the lowermost entropy generation rate obtained is 0.002736. And the optimal results are shown in Table 1: number of fin is 25; thickness of fin is 0.6mm; height of fin is 44.8mm.

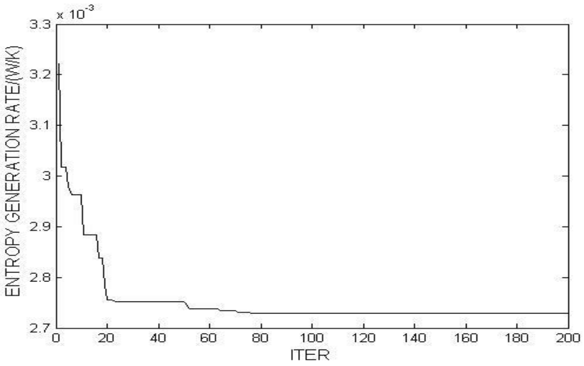

Figure 3. The convergence progress of PSO algorithm with respect to generation numbers

Table 1. Simulation result of plate fin heat sink

Number of fin	Thickness of fin [mm]	Height of fin [mm]	Base temperature [k]
25	0.6	44.8	342.6241

For the same model, we analyze the effect of fin number, fin height and fin thickness on entropy production rate. Figure 4 shows the entropy generation rate from a heat sink with respect to its different fin pitch while fin number is 25, fin thickness is 0.6mm. With the increase of fin height, entropy production rate decreases and converge. But the material wastes more. In order to have a better economy and a good efficiency of heat sink, the height of fin should lie in the range between 42mm and 50mm. Because in this range, the rate of entropy production rate decreases quickly and the material cost at a low level.

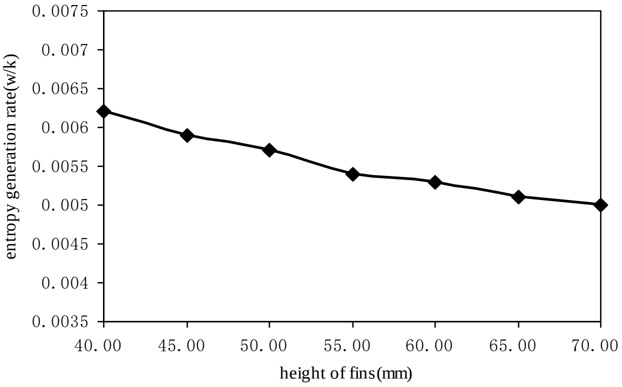

Figure 4. Entropy generation rate versus height of fins

Figure 5 shows a relationship between the entropy generation rate and the number of fins. Constraint conditions as follows: the height of fins is 44.8mm and the thickness of fins is 0.6mm. Increasing the number of fins beyond the optimized value would lead to a decrease in the entropy generation rate. Because of the convection heat transfer area increased. Continue to increase the number of fin, the increase in the fluid drag associated with fin number results in an increase in the entropy generation rate. So the number of fin should lie in the range between 22 and 28.

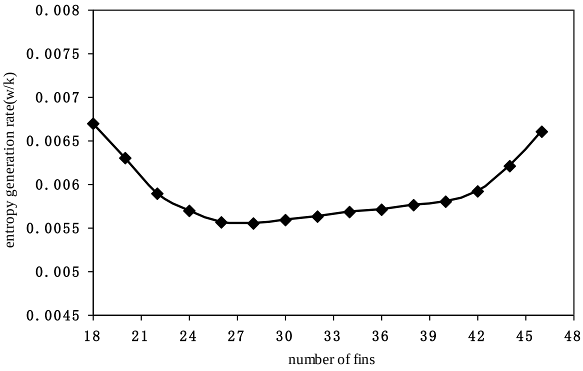

Figure 5. Entropy generation rate versus number of fins

Figure 6 shows a relationship between the entropy generation rate and the thickness of fins. Constraint conditions as follows: the height of fins is 44.8mm and the number of fins is 25. Increasing the thickness of fins beyond the optimized value would lead to a decrease in the entropy generation rate. And then, continue to increase the number of fin. Entropy generation rate would increase due to the increase of fluid drag. So, the thickness of fin should lie in the range between 0.5mm and 1.5mm.

As we can see, the results of optimization, as shown in Table 1, inside the scope described above. At the same time, the order of influence on the entropy production rate is obtained according to the amount of reduction of entropy production rate: t>n>H.

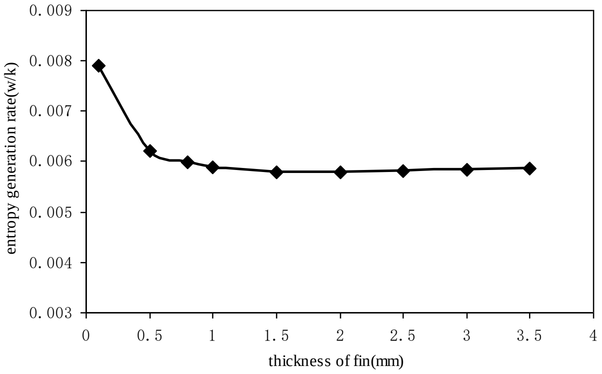

Figure 6. Entropy generation rate versus thickness of fins

7. CONCLUSIONS

The minimization of entropy generation rate as objective function is widely used on the thermodynamic optimization of plant-fin heat sink. But there is no literature reported using the particle swarm algorithm to optimize the plant-fin heat sink under natural convection. There, we established the mathematical model about a relationship between the entropy production rate and heat sink design parameters. Three design parameters as optimization variables are fin number, fin thickness and fin height. Then, we use particle swarm optimization algorithm to design the plate-fin heat sink considering the minimization of entropy generation. In this study, the constraints of the object function in particle swarm optimization (PSO) algorithm use constant penalty function method. Then, the optimal design parameters can be gained as Table1 showed: number of fin is 25; thickness of fin is 0.6mm; height of fin is 44.8mm and the corresponding temperature of the base plant is 342.6241k. The lowermost entropy generation rate obtained is 0.002736w/k. Furthermore, the effect of design variables on entropy generation rate is also presented as shown in Figure 4 to Figure 6. And we can find the most important factor affecting entropy production rate is the thickness of fin. At the same time, the present study demonstrates the particle swarm algorithm can provide a strong ability of auto-search and few in parameters in the optimization design of heat sink. And, in this paper, PSO is applied to the optimization design of the plant-fin heat sink, which can be extended to other types of heat sink. For other types of heat sink optimization, we need to do is according to different design conditions and requirements to choose the optimization objectives, constrains and optimization variables. Then based on the PSO algorithm can get the optimization results.

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NOMENCLATURE

A_c	cross section area of fin, m^2
b	spacing between two fins, m
c_1, c_2	acceleration parameter (for PSO algorithm)
constant	Penalty constant
C_p	specific heat , J/kg K
EI	Elenbaas number
g	acceleration of gravity, m/s^2
g_i	constraint
H	height of fin, m
h	heat transfer coefficient, W/m^2 K
K	thermal conductivity of heat sink, W/m K
K_a	thermal conductivity of air, W/m K
L	heat sink length, m
m	fin parameter $\approx \sqrt{hP/K_aA_c}$, m^{-1}
n	fin number
P	cross section circumference of the fin, m
px	particle’s position (for PSO algorithm)
Q	heat load, W
R_{base}	the thermal resistance of the bulk material, K/W
R_{sink}	the overall thermal resistance of the heat sink, K/W
R_{total}	the overall thermal resistance of fins, K/W
R_{fin}	thermal resistance of each fin, K/W
$\dot{S}_{gen}$	entropy generation rate, W/ K
T_1	base temperature, K
T_2	ambient temperature, K
t	thickness of fin, m
t_b	base plate thickness, m
v	particle velocity (for PSO algorithm)
w	inertia weight (for PSO algorithm)
W	width of the plate-fin, m
x_i	design variables

Greek letters

β	thermal expansion coefficient
ρ	density of air, kg/m^3
η	fin efficiency
$\bar{\theta}$	average temperature difference between heat sink and ambient air, k
θ_b	temperature excess of the heat sink base plate, k

Subscripts

a	air
b	base plate
fin	single fin
i	particle index(for PSO algorithm)
gen	generation
max	maximum
min	minimum
sink	heat sink